2102311 Electrical Measurement and Instruments (Part II)

- **Bridge Circuits (DC and AC)**
- **Electronic Instruments (Analog & Digital)**
- **Signal Generators**
- **Frequency and Time Interval Measurements**
- **Introduction to Transducers**

อาภรณ์ ธีรมงคลรัศมี
ตึกไฟฟ้า 6 ชั้น ห้อง 306

Textbook:

-A.D. Helfrick, and W.D. Cooper, "*Modern Electronic Instrumentation and Measurement Techniques*" Prentice Hall, 1994.

- D.A. Bell, "*Electronic Instrumentation and Measurements*", 2nd ed., Prentice Hell, 1994.

Resistor Types

Importance parameters

- ❖Value
- ❖Tolerance
- ❖Power rating
- ❖Temperature coefficient

Type	Values (Ω)	Power rating (W)	Tolerance (%)	Temperature coefficient (ppm/°C)	picture
Wire wound (power)	10m~3k	3~1k	±1~±10	±30~±300	
Wire wound (precision)	10m~1M	0.1~1	±0.005~±1	±3~±30	
Carbon film	1~1M	0.1~3	±2~±10	±100~±200	
Metal film	100m~1M	0.1~3	±0.5~±5	±10~±200	
Metal film (precision)	10m~100k	0.1~1	±0.05~±5	±0.4~±10	
Metal oxide film	100m~100k	1~10	±2~±10	±200~±500	

Data: Transistor technology (10/2000)

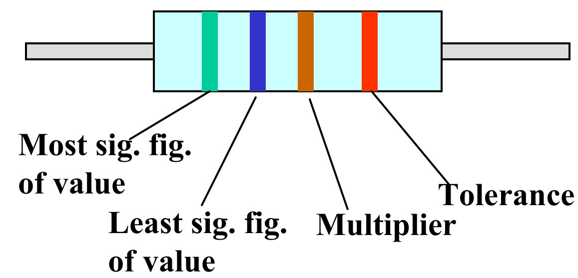
Resistor Values

- Color codes
- Alphanumeric

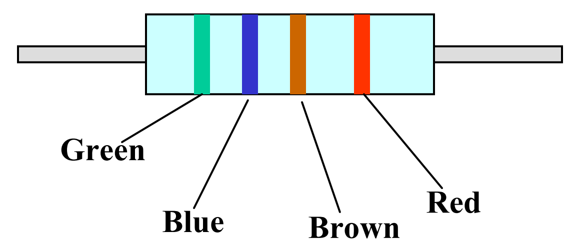
	Color	Digit	Multiplier	Tolerance (%)		Temperature coefficient (ppm/°C)	
	Silver	-	10^{-2}	±10	K	-	-
	Gold		10^{-1}	±5	J		
	Black	0	10^{0}	-	-	±250	K
	Brown	1	10^{1}	±1	F	±100	H
	Red	2	10^{2}	±2	G	±50	G
	Orange	3	10^{3}	-	-	±15	D
	Yellow	4	10^{4}		-	±25	F
	Green	5	10^{5}	±0.5	D	±20	E
	Blue	6	10^{6}	±0.25	C	±10	C
	Violet	7	10^{7}	±0.1	B	±5	B
	Gray	8	10^{8}	-	-	±1	A
	White	9	10^{9}		-	-	-
			-	±20	M		

Data: Transistor technology (10/2000)

4 band color codes



Ex.



$R = 560\ \Omega \pm 2\%$

Alphanumeric

R, K, M, G, and T =
$\times 10^{0}$, $\times 10^{3}$, $\times 10^{6}$, $\times 10^{9}$, and $\times 10^{12}$

Ex. 6M8 = $6.8 \times 10^{6}\ \Omega$
59P04 = $59.04\ \Omega$

Resistor Values

$$R = x \pm \%\Delta x$$

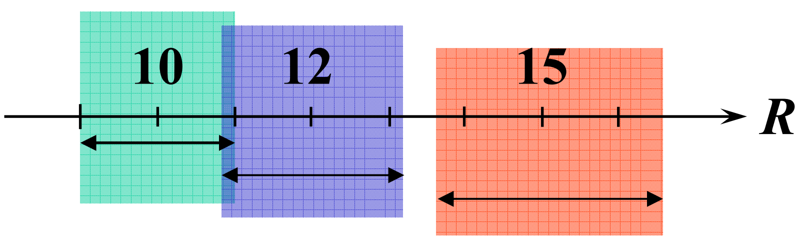
Tolerance

Nominal value

Ex. 1 kΩ ± 10% ≡ 900-1100 Ω

For 10% resistor

10, 12, 15, 18, …



$$R \approx \sqrt[E]{10^n}$$

where E = 6, 12, 24, 96

for 20, 10, 5, 1% tolerance

n = 0, 1, 2, 3, …

For 10% resistor E = 12

$n = 0$; $R = 1.00000\ldots$

$n = 1$; $R = 1.21152\ldots$

$n = 2$; $R = 1.46779\ldots$

$n = 3$; $R = 1.77827\ldots$

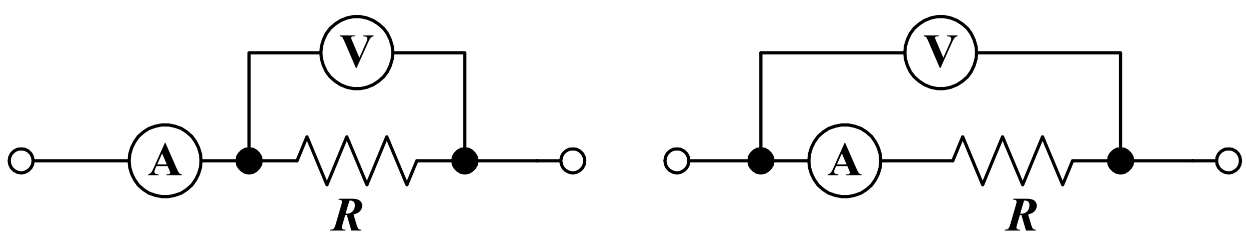
Commonly available resistance for a fixed resistor

±1%	±2%	±5%	±10%	±1%	±2%	±5%	±10%
100	100	10	10	316	316		
102				324			
105	105			332	332	33	33
107				340			
110	110	11		348	348		
113				357			
115	115			365	365	36	
118				374			
121	121	12	12	383	383		
124				392		39	39
127	127			407	407		
130		13		412			
133	133			422	422		
137				432		43	
140	140			442	442		
143				453			
147	147			464	464		
150		15	15	475		47	47
154	154			487	487		
158				499			
162	162	16		511	511	51	
165				523			
169	169			536	536		
174				549			
178	178			562	562	56	56
182		18	18	576			
187	187			590	590		
191				604			
196	196			619	619	62	
200		20		634			
205	205			649	649		
210				665			
215	215			681	681	68	68
221		22	22	698			
226	226			715	715		
232				732			
237	237			750	750	75	
243		24		765			
249	249			787	787		
255				806			
261	261			825	825	82	82
267				845			
274	274	27	27	866	866		
280				887			
287	287			909	909	91	
294				931			
301	301	30		953	953		
309				976			

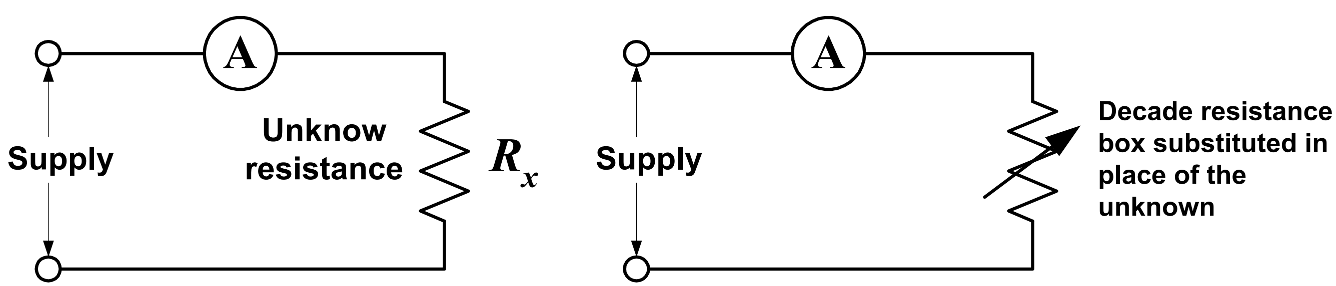
Resistance Measurement Techniques

- Bridge circuit
- Voltmeter-ammeter
- Substitution
- Ohmmeter

Voltmeter-ammeter



Substitution



Voltmeter-ammeter method

Pro and con:

- **Simple and theoretical oriented**
- **Requires two meter and calculations**
- **Subject to error: Voltage drop in ammeter (Fig. (a))**
  Current in voltmeter (Fig. (b))

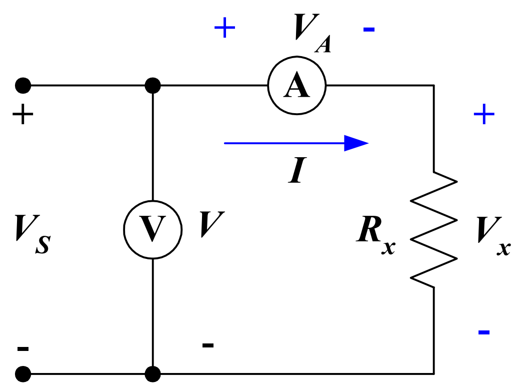


Fig. (a)

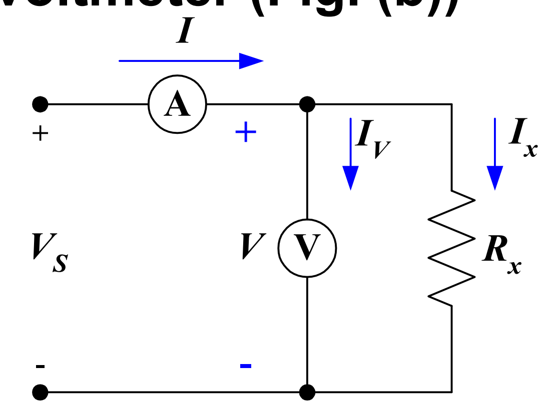


Fig. (b)

Measured R_x: $R_{\text{meas}} = \dfrac{V}{I} = \dfrac{V_x + V_A}{I} = R_x + \dfrac{V_A}{I}$

if $V_x >> V_A$ $\qquad R_{\text{meas}} \approx R_x$

Therefore this circuit is suitable for measure large resistance

Measured R_x: $R_{\text{meas}} = \dfrac{V}{I} = \dfrac{V}{I_x + I_V} = \dfrac{R_x}{1 + I_V / I_x}$

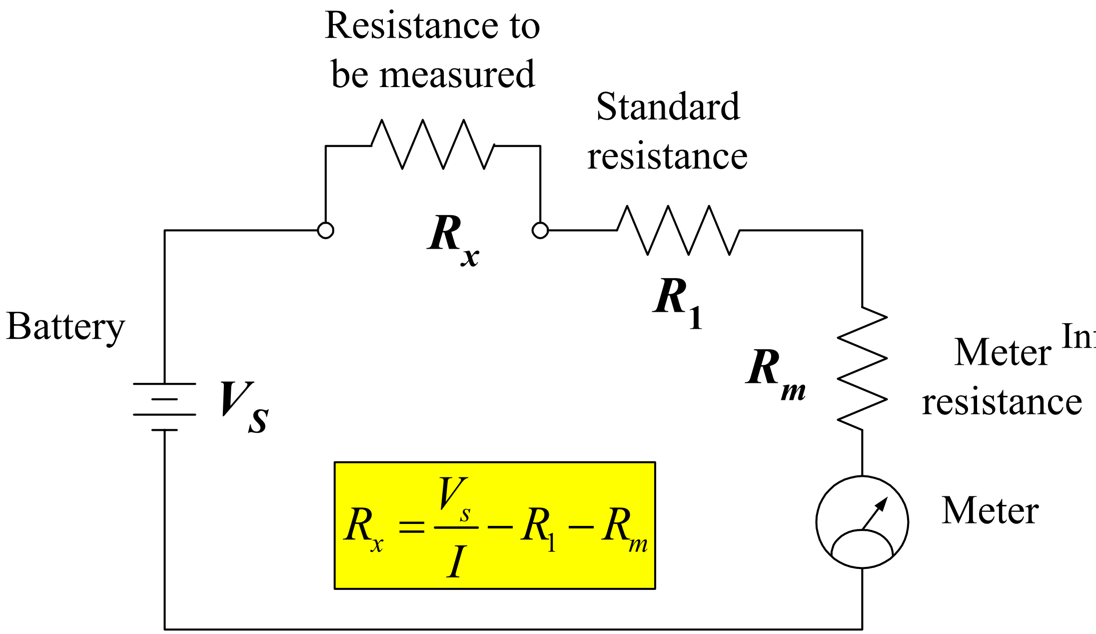
if $I_x >> I_V$ $\qquad R_{\text{meas}} \approx R_x$

Therefore this circuit is suitable for measure small resistance

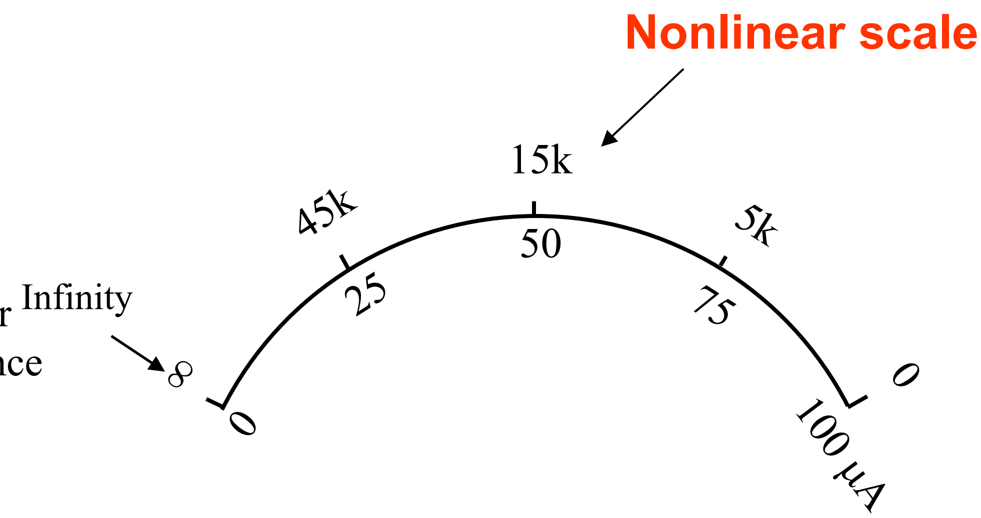
Ohmmeter

- Voltmeter-ammeter method is rarely used in practical applications (mostly used in Laboratory)
- Ohmmeter uses only one meter by keeping one parameter constant

Example: series ohmmeter



Basic series ohmmeter



Ohmmeter scale

Basic series ohmmeter consisting of a PMMC and a series-connected standard resistor (R_1). When the ohmmeter terminals are shorted ($R_x = 0$) meter full scale defection occurs. At half scale defection $R_x = R_1 + R_m$, and at zero defection the terminals are open-circuited.

Bridge Circuit

Bridge Circuit is a null method, operates on the principle of comparison. That is a known (standard) value is adjusted until it is equal to the unknown value.

Bridge Circuit

DC Bridge (Resistance)

- Wheatstone Bridge
- Kelvin Bridge
- Megaohm Bridge

AC Bridge

Inductance

- Maxwell Bridge
- Hay Bridge
- Owen Bridge
- Etc.

Capacitance

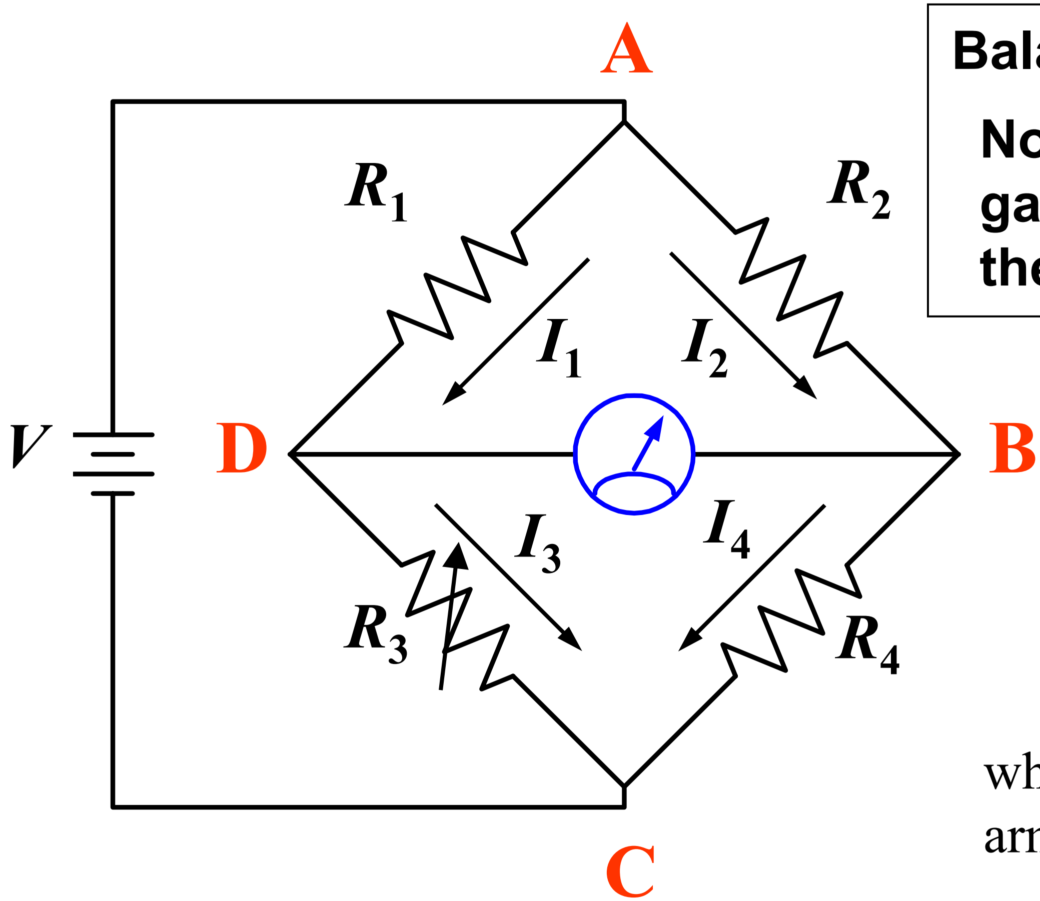
- Schering Bridge

Frequency

- Wien Bridge

Wheatstone Bridge and Balance Condition

Suitable for moderate resistance values: 1 Ω to 10 MΩ



Balance condition:

No potential difference across the galvanometer (there is no current through the galvanometer)

Under this condition: $V_{AD} = V_{AB}$

$$I_1R_1 = I_2R_2$$

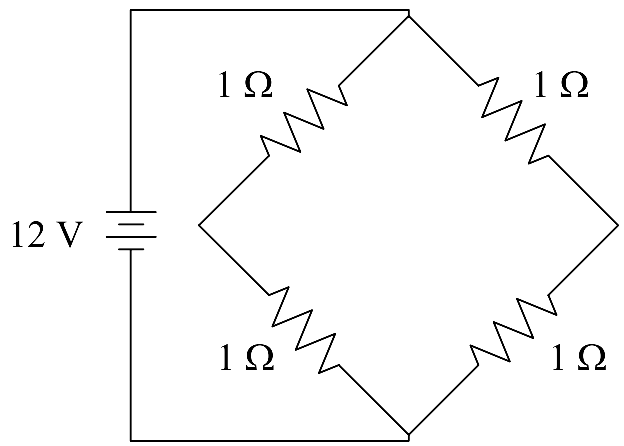
And also $V_{DC} = V_{BC}$

$$I_3R_3 = I_4R_4$$

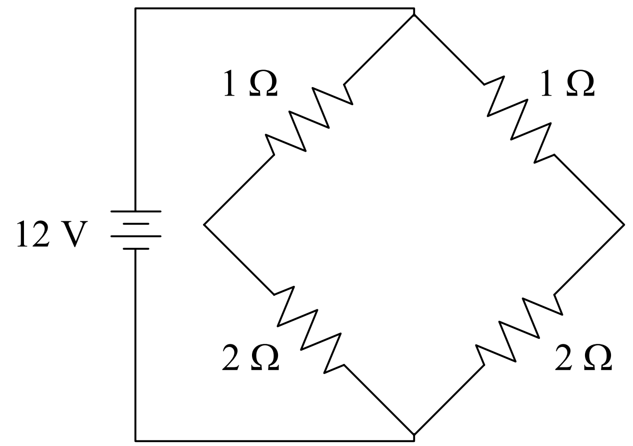
where I_1, I_2, I_3, and I_4 are current in resistance arms respectively, since $I_1 = I_3$ and $I_2 = I_4$

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \quad \text{or} \quad \boxed{R_x = R_4 = R_3\frac{R_2}{R_1}}$$

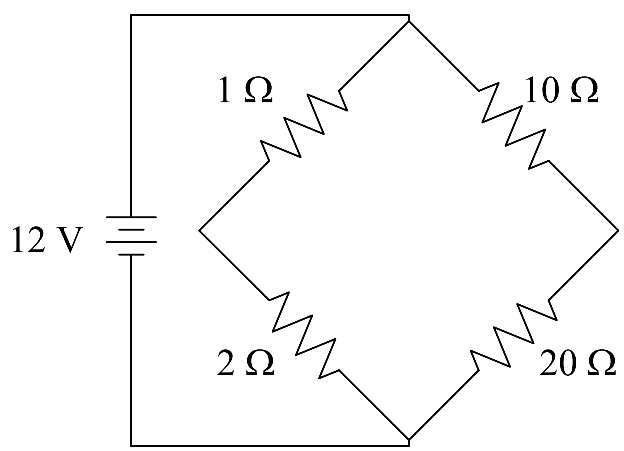
Example



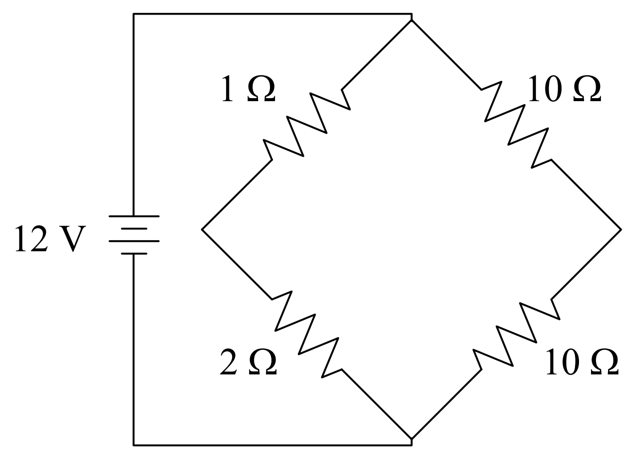
(a) Equal resistance



(b) Proportional resistance



(c) Proportional resistance



(d) 2-Volt unbalance

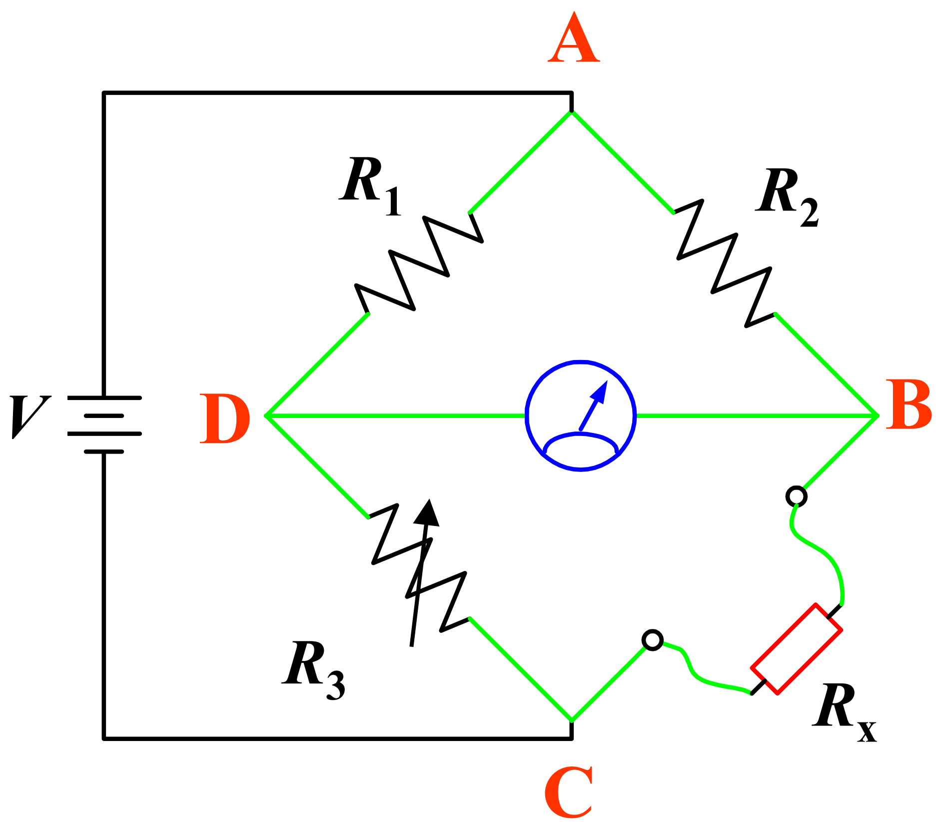
Measurement Errors

1. Limiting error of the known resistors

$$R_x = (R_3 \pm \Delta R_3)\left(\frac{R_2 \pm \Delta R_2}{R_1 \pm \Delta R_1}\right)$$

Using 1st order approximation:

$$\boxed{R_x = R_3 \frac{R_2}{R_1}\left(1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3}\right)}$$



2. Insufficient sensitivity of Detector

3. Changes in resistance of the bridge arms due to the heating effect (I^2R) or temperatures

4. Thermal emf or contact potential in the bridge circuit

5. Error due to the lead connection

3, 4 and 5 play the important role in the measurement of low value resistance

Example In the Wheatstone bridge circuit, R_3 is a decade resistance with a specified in accuracy ±0.2% and R_1 and R_2 = 500 Ω ± 0.1%. If the value of R_3 at the null position is 520.4 Ω, determine the possible minimum and maximum value of R_X

SOLUTION Apply the error equation
$$\boxed{R_x = R_3 \frac{R_2}{R_1}\left(1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3}\right)}$$

$$R_x = \frac{520.4 \times 500}{500}\left(1 \pm \frac{0.1}{100} \pm \frac{0.1}{100} \pm \frac{0.2}{100}\right) = 520.4(\ 1 \pm 0.004) = 520.4 \pm 0.4\%$$

Therefore the possible values of R_3 are 518.32 to 522.48 Ω

Example A Wheatstone bridge has a ratio arm of 1/100 (R_2/R_1). At first balance, R_3 is adjusted to 1000.3 Ω. The value of R_x is then changed by the temperature change, the new value of R_3 to achieve the balance condition again is 1002.1 Ω. Find the change of R_x due to the temperature change.

SOLUTION At first balance:
$$R_x\text{old} = R_3 \frac{R_2}{R_1} = 1000.3 \times \frac{1}{100} = 10.003\ \Omega$$

After the temperature change:
$$R_x\text{new} = R_3 \frac{R_2}{R_1} = 1002.1 \times \frac{1}{100} = 10.021\ \Omega$$

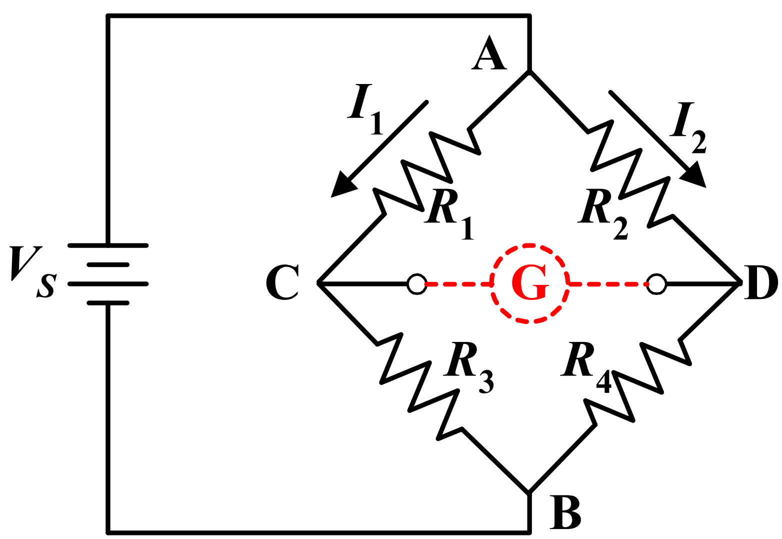
Therefore, the change of R_x due to the temperature change is 0.018 Ω

Sensitivity of Galvanometer

A galvanometer is use to detect an unbalance condition in Wheatstone bridge. Its sensitivity is governed by: *Current sensitivity (currents per unit defection) and internal resistance.*

consider a bridge circuit under a small unbalance condition, and apply circuit analysis to solve the current through galvanometer

Thévenin Equivalent Circuit



Thévenin Voltage (V_{TH})

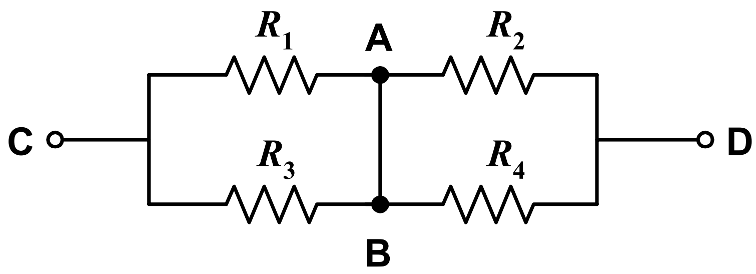
$$V_{CD} = V_{AC} - V_{AD} = I_1 R_1 - I_2 R_2$$

where $I_1 = \dfrac{V}{R_1 + R_3}$ and $I_2 = \dfrac{V}{R_2 + R_4}$

Therefore $$\boxed{V_{TH} = V_{CD} = V\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right)}$$

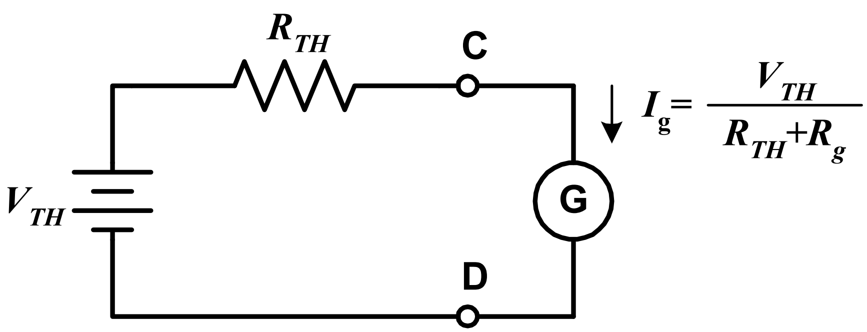
Sensitivity of Galvanometer (continued)

Thévenin Resistance (R_{TH})



$$R_{TH} = R_1 // R_3 + R_2 // R_4$$

Completed Circuit

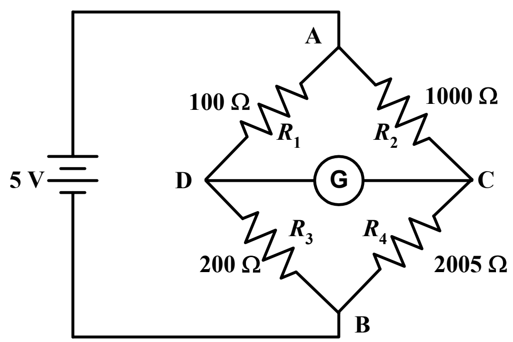


$$I_g = \frac{V_{TH}}{R_{TH} + R_g}$$

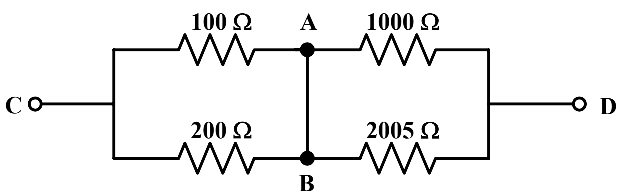
where I_g = the galvanometer current
R_g = the galvanometer resistance

Example 1 Figure below show the schematic diagram of a Wheatstone bridge with values of the bridge elements. The battery voltage is 5 V and its internal resistance negligible. The galvanometer has a current sensitivity of 10 mm/μA and an internal resistance of 100 Ω. Calculate the deflection of the galvanometer caused by the 5-Ω unbalance in arm *BC*

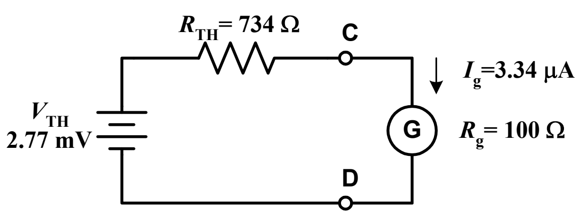
SOLUTION The bridge circuit is in the small unbalance condition since the value of resistance in arm BC is 2,005 Ω.



(a)



(b)



(c)

Thévenin Voltage (V_{TH})

$$V_{TH} = V_{AD} - V_{AC} = 5\text{ V} \times \left(\frac{100}{100+200} - \frac{1000}{1000+2005} \right)$$

$$\approx 2.77\text{ mV}$$

Thévenin Resistance (R_{TH})

$$R_{TH} = 100 // 200 + 1000 // 2005 = 734\ \Omega$$

The galvanometer current

$$I_g = \frac{V_{TH}}{R_{TH} + R_g} = \frac{2.77\text{ mV}}{734\ \Omega + 100\ \Omega} = 3.32\ \mu\text{A}$$

Galvanometer deflection

$$d = 3.32\ \mu\text{A} \times \frac{10\text{ mm}}{\mu\text{A}} = 33.2\text{ mm}$$

Example 2 The galvanometer in the previous example is replaced by one with an internal resistance of 500 Ω and a current sensitivity of 1mm/μA. Assuming that a deflection of 1 mm can be observed on the galvanometer scale, determine if this new galvanometer is capable of detecting the 5-Ω unbalance in arm *BC*

SOLUTION Since the bridge constants have not been changed, the equivalent circuit is again represented by a Thévenin voltage of 2.77 mV and a Thévenin resistance of 734 Ω. The new galvanometer is now connected to the output terminals, resulting a galvanometer current.

$$I_g = \frac{V_{TH}}{R_{TH} + R_g} = \frac{2.77 \text{ mV}}{734\ \Omega + 500\ \Omega} = 2.24\ \mu\text{A}$$

The galvanometer deflection therefore equals 2.24 μA x 1 mm/μA = 2.24 mm, indicating that this galvanometer produces a deflection that can be easily observed.

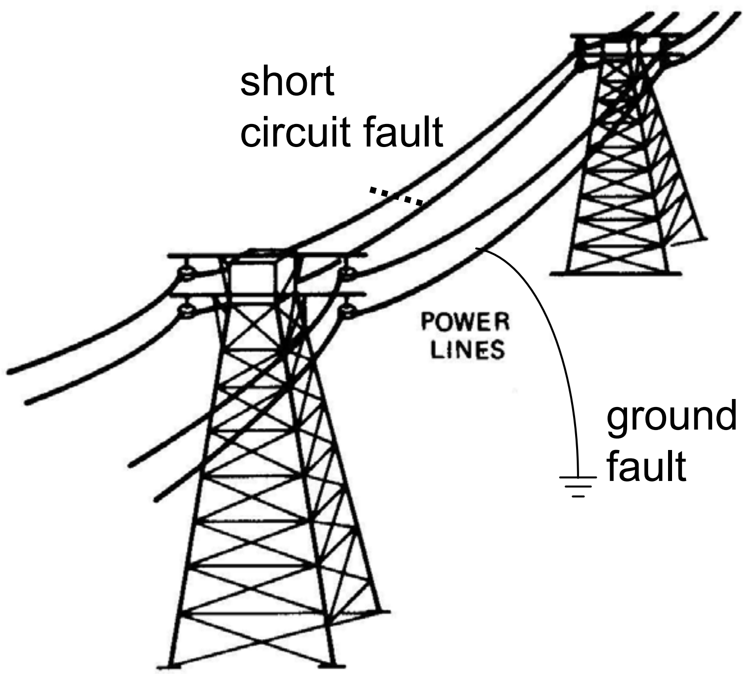
Example 3 If all resistances in the Example 1 increase by 10 times, and we use the galvanometer in the Example 2. Assuming that a deflection of 1 mm can be observed on the galvanometer scale, determine if this new setting can be detected (the 50-Ω unbalance in arm *BC*)

SOLUTION

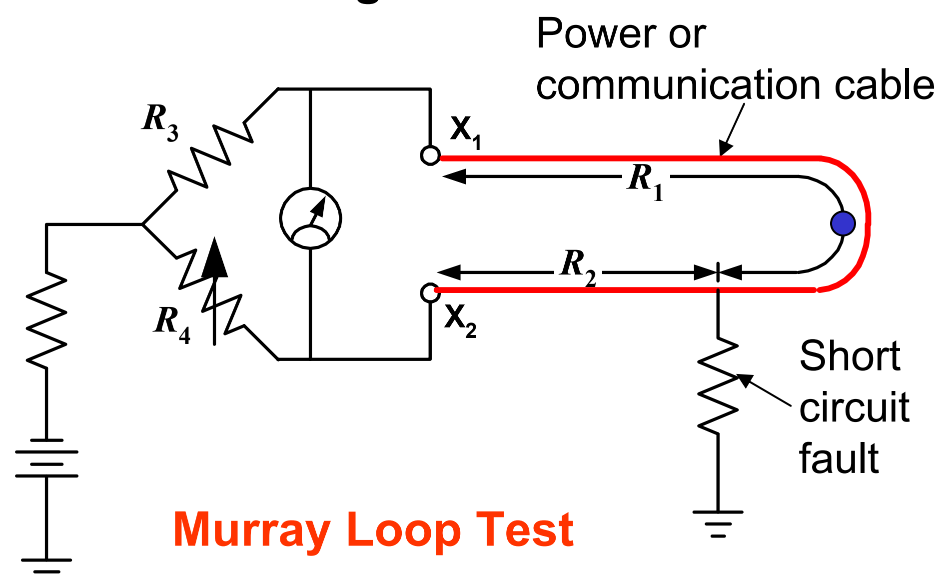
Application of Wheatstone Bridge

Murray/Varrley Loop Short Circuit Fault (Loop Test)

- **Loop test can be carried out for the location of either a ground or a short circuit fault.**



Assume: earth is a good conductor



Murray Loop Test

Let $R = R_1 + R_2$

At balance condition: $\dfrac{R_3}{R_4} = \dfrac{R_1}{R_2}$

$$\boxed{R_1 = R\left(\frac{R_3}{R_3 + R_4}\right)} \qquad \boxed{R_2 = R\left(\frac{R_4}{R_3 + R_4}\right)}$$

The value of R_1 and R_2 are used to calculate back into distance.

Murray/Varrley Loop Short Circuit Fault (Loop Test)

Examples of commonly used cables (Approx. R at 20°C)

Wire dia. In mm	Ohms per km.	Meter per ohm
0.32	218.0	4.59
0.40	136.0	7.35
0.50	84.0	11.90
0.63	54.5	18.35
0.90	27.2	36.76

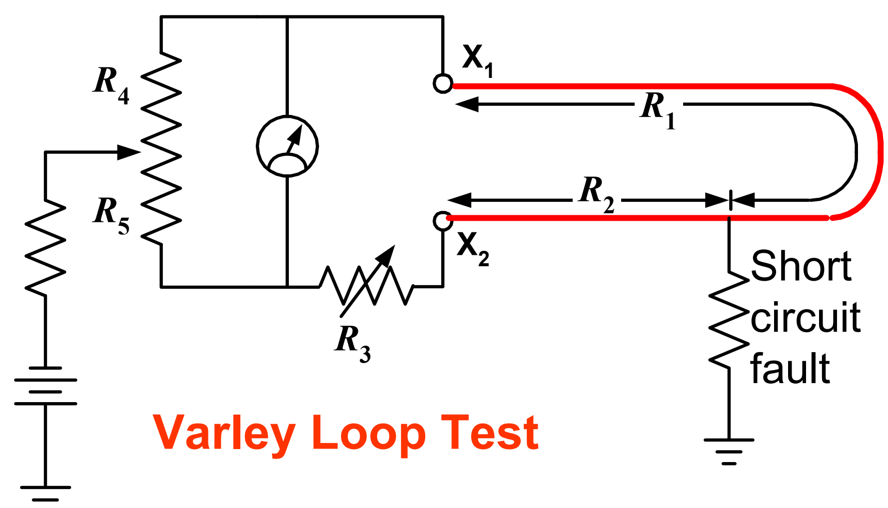
Remark The resistance of copper increases 0.4% for 1°C rise in Temp.

Let $\boldsymbol{R} = \boldsymbol{R_1} + \boldsymbol{R_2}$ and define Ratio $= \boldsymbol{R_4}/\boldsymbol{R_5}$

At balance condition: $\text{Ratio} = \dfrac{R_4}{R_5} = \dfrac{R_1}{R_2 + R_3}$

$$R_1 = \frac{\text{Ratio}}{\text{Ratio}+1} R + R_3$$

$$R_2 = \frac{R - \text{Ratio} R_3}{\text{Ratio}+1}$$

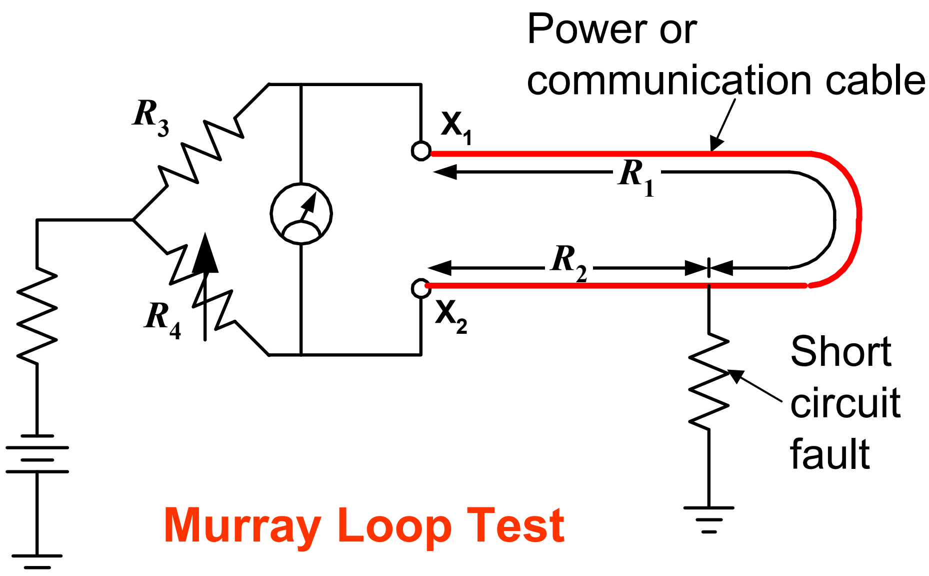


Varley Loop Test

Example Murray loop test is used to locate ground fault in a telephone system. The total resistance, $R = R_1 + R_2$ is measured by Wheatstone bridge, and its value is 300 Ω. The conditions for Murray loop test are as follows:

$$R_3 = 1000\ \Omega \text{ and } R_4 = 500\ \Omega$$

Find the location of the fault in meter, if the length per Ohm is 36.67 m.



SOLUTION

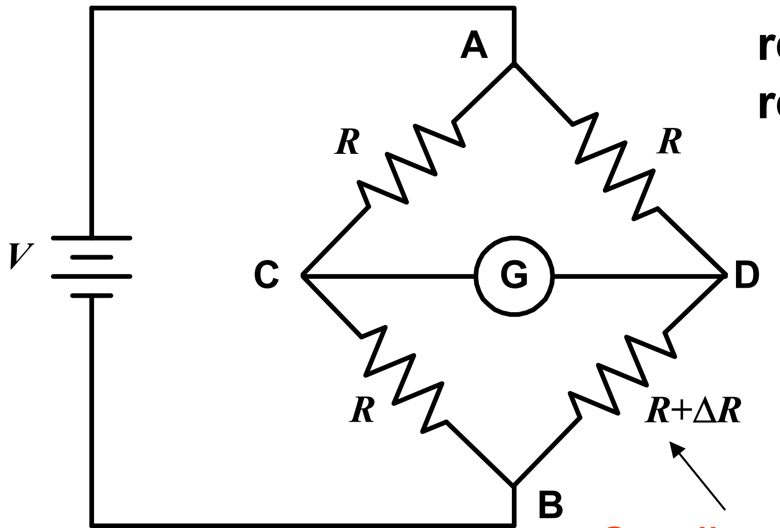
$$R_1 = R\left(\frac{R_3}{R_3 + R_4}\right) = 300 \times \frac{1000}{1000 + 500} = 200\ \Omega$$

$$R_2 = R\left(\frac{R_4}{R_3 + R_4}\right) = 300 \times \frac{500}{1000 + 500} = 100\ \Omega$$

Therefore, the location from the measurement point is $100\ \Omega \times 36.67\ \text{m}/\Omega = 3667\ \text{m}$

Application of Wheatstone Bridge

Unbalance bridge



Small unbalance occur by the external environment

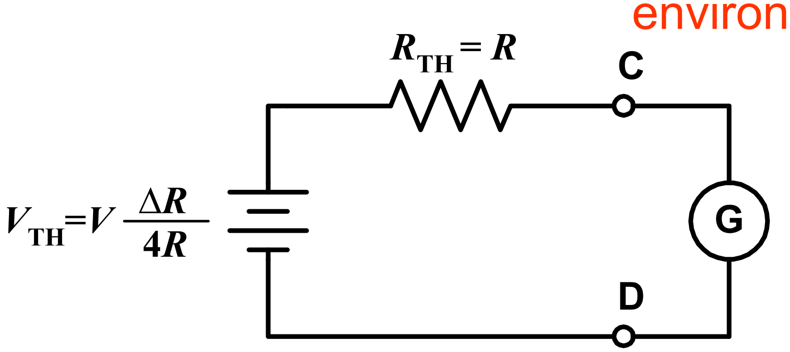
Consider a bridge circuit which have identical resistors, R in three arms, and the last arm has the resistance of $R + \Delta R$. if $\Delta R / R << 1$

Thévenin Voltage (V_{TH})

$$\boxed{V_{TH} = V_{CD} \approx V \frac{\Delta R}{4R}}$$

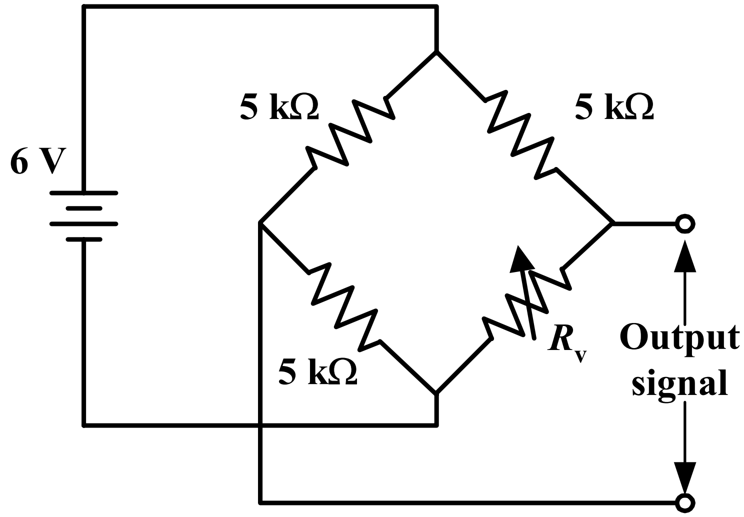
Thévenin Resistance (R_{TH})

$$\boxed{R_{TH} \approx R}$$

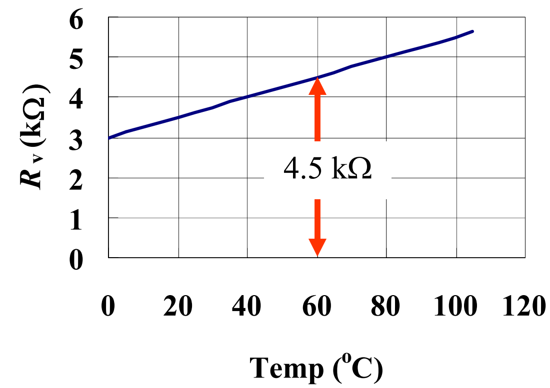


This kind of bridge circuit can be found in sensor applications, where the resistance in one arm is sensitive to a physical quantity such as pressure, temperature, strain etc.

Example Circuit in Figure (a) below consists of a resistor R_v which is sensitive to the temperature change. The plot of R VS $Temp$. is also shown in Figure (b). Find (a) the temperature at which the bridge is balance and (b) The output signal at Temperature of 60°C.



(a)



(b)

SOLUTION (a) at bridge balance, we have $R_v = \dfrac{R_3 \times R_2}{R_1} = \dfrac{5\text{ k}\Omega \times 5\text{ k}\Omega}{5\text{ k}\Omega} = 5\text{ k}\Omega$

The value of R_v = 5 kΩ corresponding to the temperature of 80°C in the given plot.

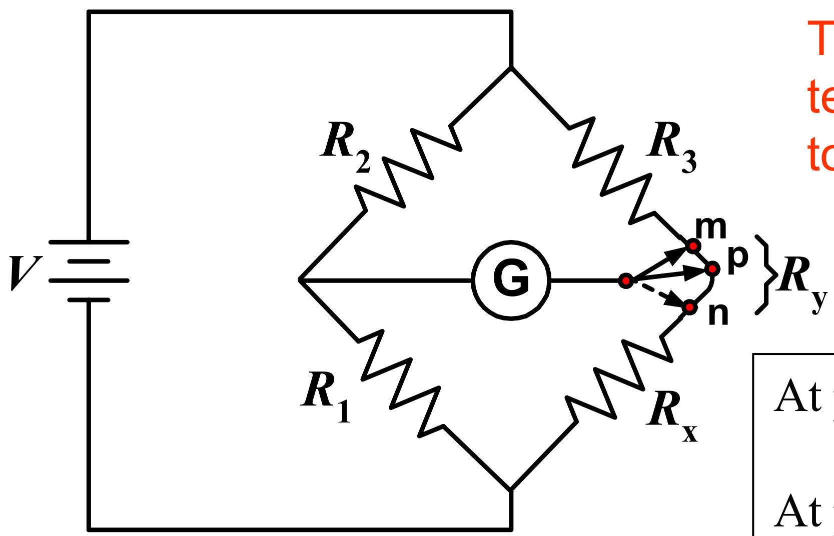
(b) at temperature of 60°C, R_v is read as 4.5 kΩ, thus $\Delta R = 5 - 4.5 = 0.5$ kΩ. We will use Thévenin equivalent circuit to solve the above problem.

$$V_{TH} = V\frac{\Delta R}{4R} = 6\text{ V} \times \frac{0.5\text{ k}\Omega}{4 \times 5\text{ k}\Omega} = 0.15\text{ V}$$

It should be noted that $\Delta R =$ 0.5 kΩ in the problem does not satisfy the assumption $\Delta R/R \ll 1$, the exact calculation gives $V_{TH} = 0.158$ V. However, the above calculation still gives an acceptable solution.

Low resistance Bridge: $R_x < 1\ \Omega$

Effect of connecting lead



The effects of the connecting lead and the connecting terminals are prominent when the value of R_x decreases to a few Ohms

R_y = the resistance of the connecting lead from R_3 to R_x

At point m: R_y is added to the unknown R_x, resulting in too high and indication of R_x

At point n: R_y is added to R_3, therefore the measurement of R_x will be lower than it should be.

At point p: $$R_x + R_{np} = \left(R_3 + R_{mp}\right)\frac{R_1}{R_2}$$

rearrange $$R_x = R_3\frac{R_1}{R_2} + R_{mp}\frac{R_1}{R_2} - R_{np}$$

Where R_{mp} and R_{np} are the lead resistance from m to p and n to p, respectively.

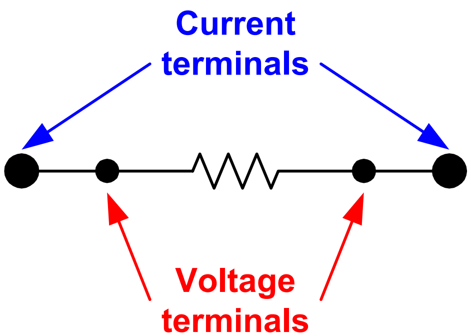
The effect of the connecting lead will be canceled out, if the sum of 2nd and 3rd term is zero.

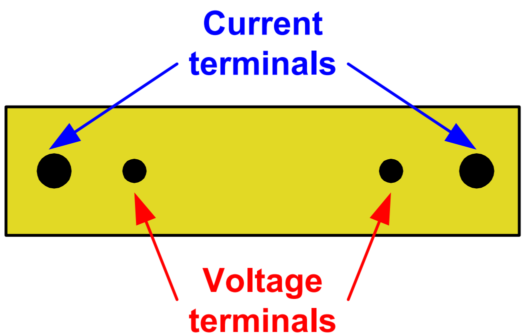
$$R_{mp}\frac{R_1}{R_2} - R_{np} = 0 \text{ or } \frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2}$$

$$\boxed{R_x = R_3\frac{R_1}{R_2}}$$

Kelvin Double Bridge: 1 to 0.00001 Ω

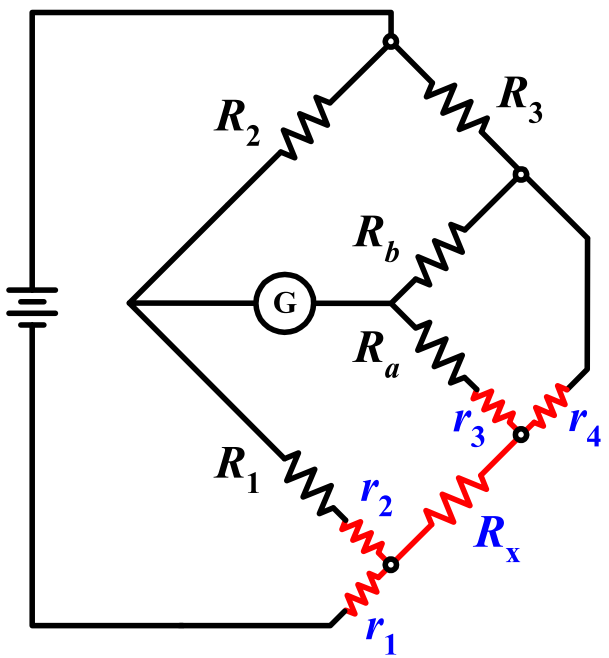
Four-Terminal Resistor





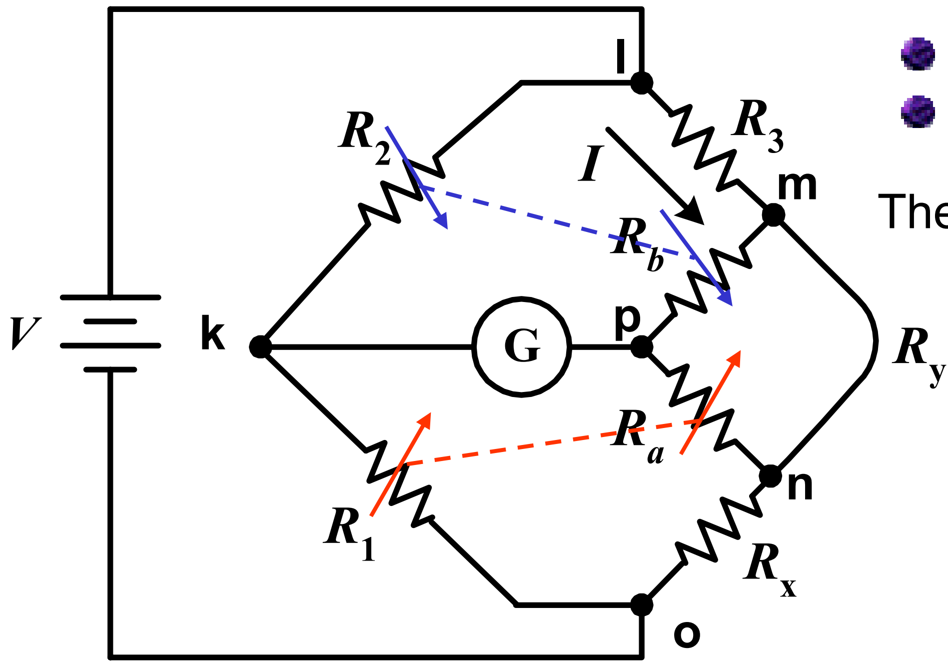
Four-terminal resistors have current terminals and potential terminals. The resistance is defined as that between the potential terminals, so that contact voltage drops at the current terminals do not introduce errors.

Four-Terminal Resistor and Kelvin Double Bridge



- r_1 causes no effect on the balance condition.
- The effects of r_2 and r_3 could be minimized, if $R_1 >> r_2$ and $R_a >> r_3$.
- The main error comes from r_4, even though this value is very small.

Kelvin Double Bridge: 1 to 0.00001 Ω



- 2 ratio arms: R_1-R_2 and R_a-R_b
- the connecting lead between m and n: yoke

The balance conditions: $V_{lk} = V_{lmp}$ or $V_{ok} = V_{onp}$

$$V_{lk} = \frac{R_2}{R_1 + R_2} V \qquad (1)$$

here $V = IR_{lo} = I[R_3 + R_x + (R_a + R_b) // R_y]$

$$V_{lmp} = I\left[R_3 + \frac{R_y}{R_a + R_b + R_y} R_b\right] \qquad (2)$$

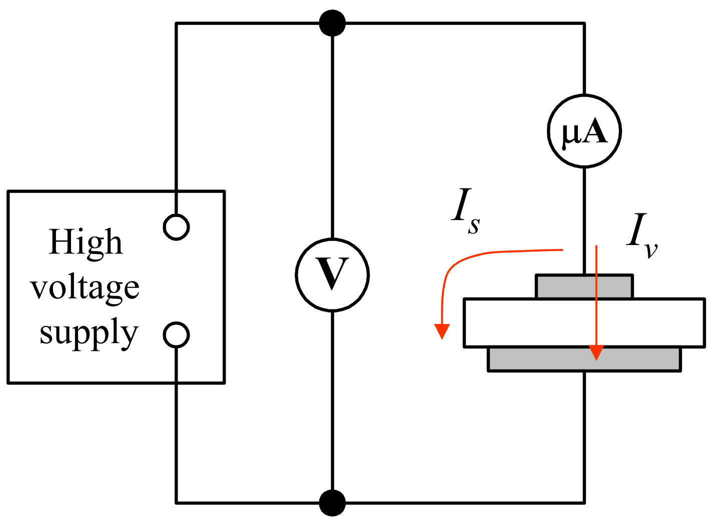
Eq. (1) = (2) and rearrange: $R_x = R_3 \frac{R_1}{R_2} + \frac{R_b R_y}{R_a + R_b + R_y}\left(\frac{R_1}{R_2} - \frac{R_a}{R_b}\right)$ ⇒ $\boxed{R_x = R_3 \frac{R_1}{R_2}}$

If we set $R_1/R_2 = R_a/R_b$, the second term of the right hand side will be zero, the relation reduce to the well known relation. In summary, The resistance of the yoke has no effect on the measurement, if the two sets of ratio arms have equal resistance ratios.

High Resistance Measurement

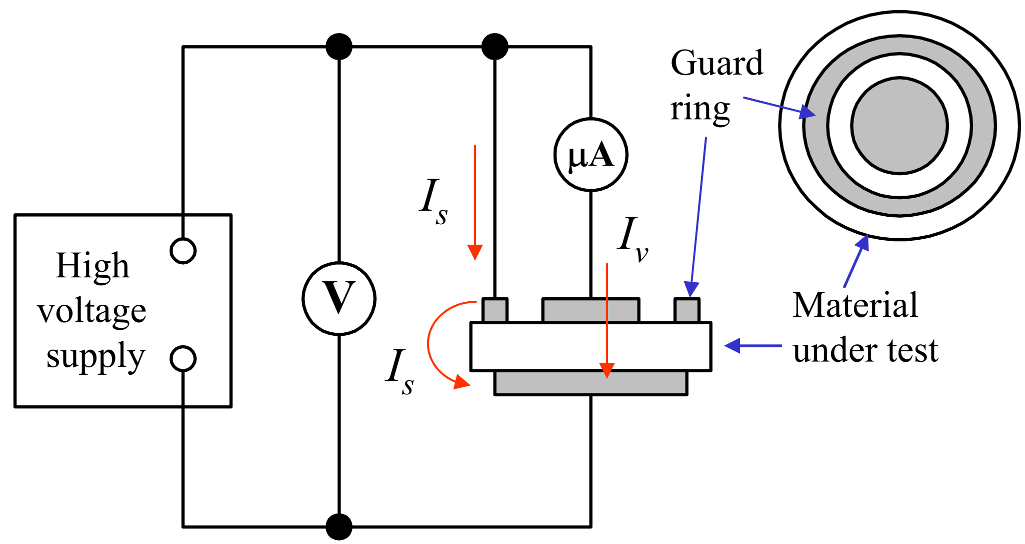
Guard ring technique:

- Volume resistance, R_V
- Surface leakage resistance, R_s



(a) Circuit that measures insulation volume resistance in parallel with surface leakage resistance

$$R_{meas} = R_s // R_v = \frac{V}{I_s + I_v}$$



(b) Use of guard ring to measure only volume resistance

$$R_{meas} = R_v = \frac{V}{I_v}$$

High Resistance Measurement

Example The Insulation of a metal-sheath electrical cable is tested using 10,000 V supply and a microammeter. A current of 5 μA is measured when the components are connected without guard wire. When the circuit is connect with guard wire, the current is 1.5 μA. Calculate (a) the volume resistance of the cable insulation and (b) the surface leakage resistance

SOLUTION

(a) Volume resistance:

$$I_V = 1.5\ \mu\text{A}$$

$$R_V = \frac{V}{I_V} = \frac{10000\ \text{V}}{1.5\ \mu\text{A}} = 6.7\times10^{9}\ \Omega$$

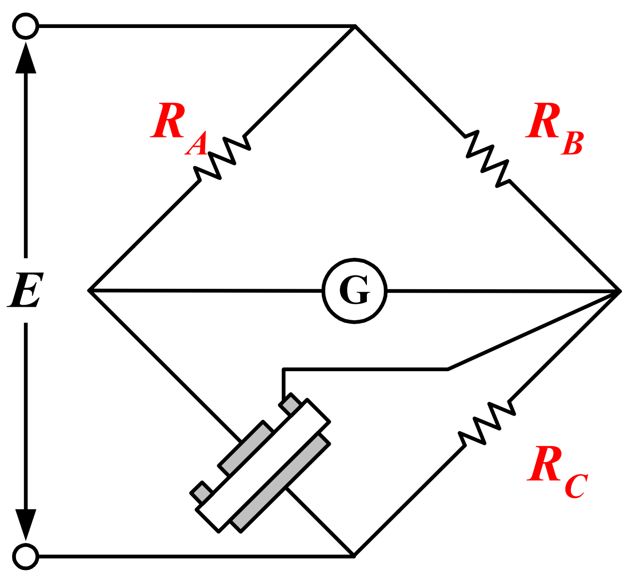
(b) Surface leakage resistance:

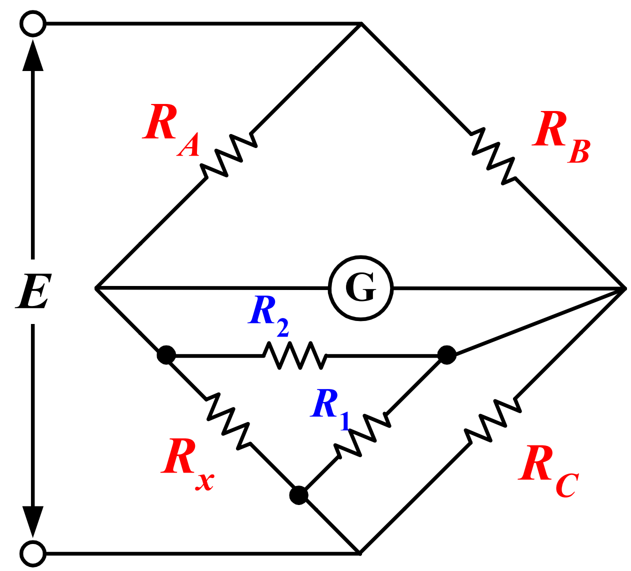
$$I_V + I_S = 5\ \mu\text{A} \qquad I_S = 5\ \mu\text{A} - I_V = 3.5\ \mu\text{A}$$

$$R_S = \frac{V}{I_S} = \frac{10000\ \text{V}}{3.5\ \mu\text{A}} = 2.9\times10^{9}\ \Omega$$

MegaOhm Bridge

- Just as low-resistance measurements are affected by series lead impedance, high-resistance measurements are affected by shunt-leakage resistance.



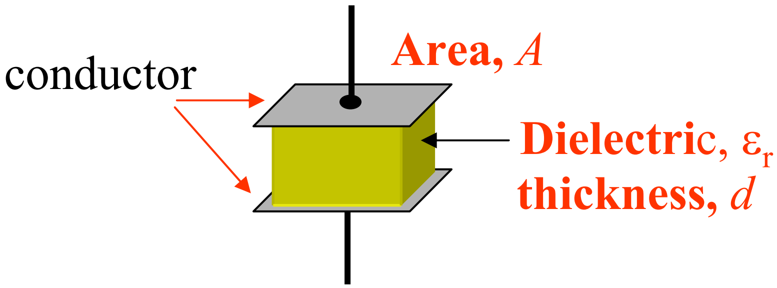


- the guard terminal is connect to a bridge corner such that the leakage resistances are placed across bridge arm with low resistances

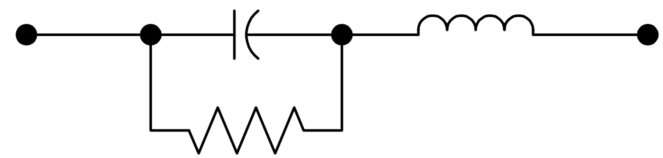
$$\left.\begin{aligned} &R_1 // R_C \approx R_C \quad \text{since } R_1 >> R_C \\ &R_2 // R_g \approx R_g \quad \text{since } R_2 >> R_g \end{aligned}\right\} \qquad \boxed{R_x \approx R_A \frac{R_C}{R_B}}$$

Capacitor

Capacitance – the ability of a dielectric to store electrical charge per unit voltage



$$C = \frac{A\,\varepsilon_0 \varepsilon_r}{d}$$

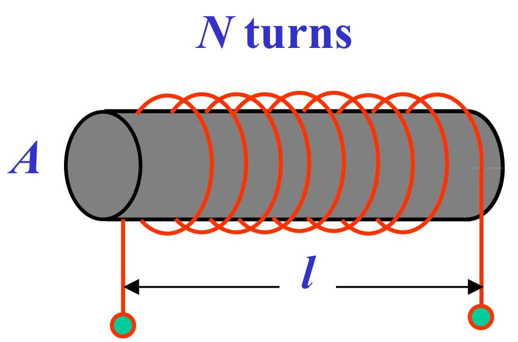


Typical values pF, nF or μF

Dielectric	Construction	Capacitance	Breakdown,V
Air	Meshed plates	10-400 pF	100 (0.02-in air gap)
Ceramic	Tubular	0.5-1600 pF	500-20,000
	Disk	1pF to 1 μF	
Electrolytic	Aluminum	1-6800 μF	10-450
	Tantalum	0.047 to 330 μF	6-50
Mica	Stacked sheets	10-5000 pF	500-20,000
Paper	Rolled foil	0.001-1 μF	200-1,600
Plastic film	Foil or Metallized	100 pF to 100 μF	50-600

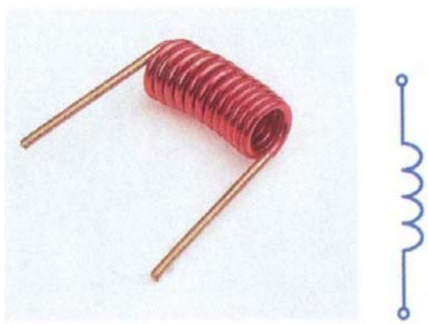
Inductor

Inductance – the ability of a conductor to produce induced voltage when the current varies.



$$L = \frac{\mu_o \mu_r N^2 A}{l}$$

$\mu_o = 4\pi\times10^{-7}$ H/m

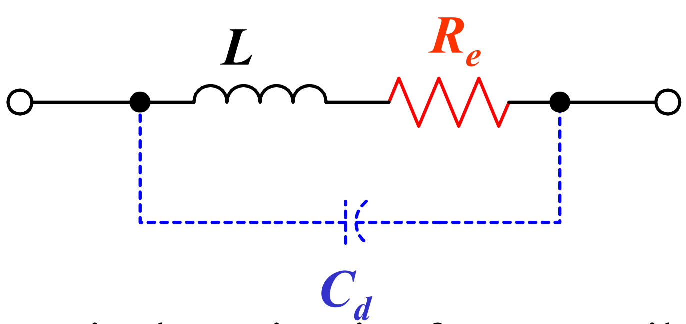


Air core inductor

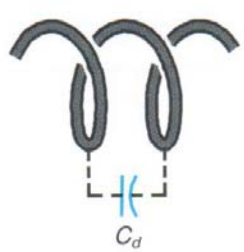
μ_r – relative permeability of core material

Ni ferrite: $\mu_r > 200$

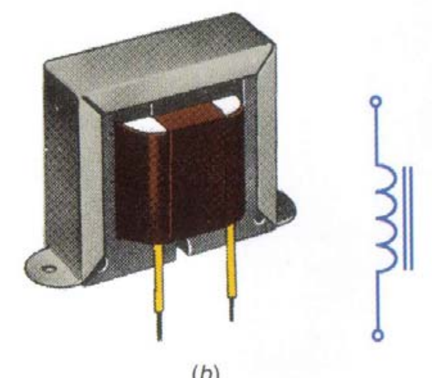
Mn ferrite: $\mu_r > 2{,}000$



Equivalent circuit of an RF coil



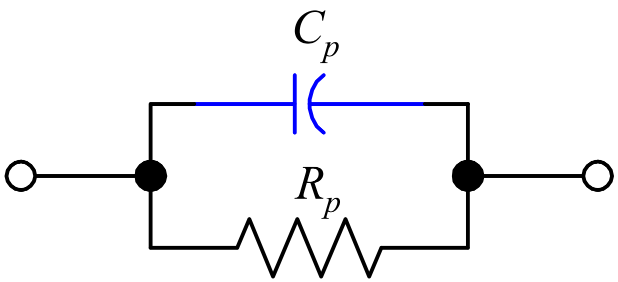
Distributed capacitance C_d between turns



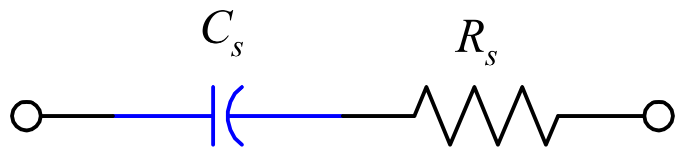
Iron core inductor

Quality Factor of Inductor and Capacitor

Equivalent circuit of capacitance

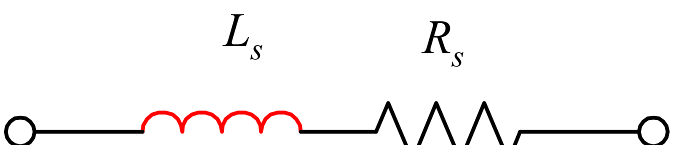


Parallel equivalent circuit

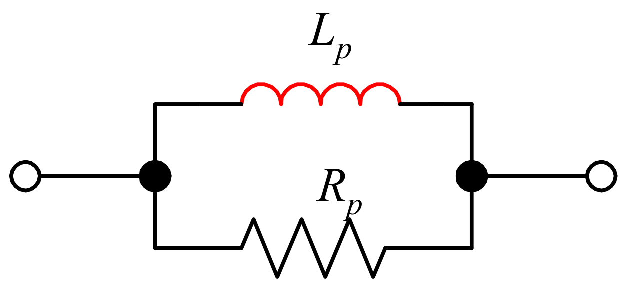


Series equivalent circuit

Equivalent circuit of Inductance



Series equivalent circuit



Parallel equivalent circuit

$$R_p = \frac{R_s^2 + X_s^2}{R_s}$$

$$X_p = \frac{R_s^2 + X_s^2}{X_s}$$

$$R_s = \frac{R_p X_p^2}{R_p^2 + X_p^2}$$

$$X_s = \frac{X_p R_p^2}{R_p^2 + X_p^2}$$

Quality Factor of Inductor and Capacitor

Quality factor of a coil: the ratio of reactance to resistance (frequency dependent and circuit configuration)

Inductance series circuit: $$Q = \frac{X_s}{R_s} = \frac{\omega L_s}{R_s}$$

Typical $Q \sim 5 – 1000$

Inductance parallel circuit: $$Q = \frac{R_p}{X_p} = \frac{R_p}{\omega L_p}$$

Dissipation factor of a capacitor: the ratio of reactance to resistance (frequency dependent and circuit configuration)

Capacitance parallel circuit: $$D = \frac{X_p}{R_p} = \frac{1}{\omega C_p R_p}$$

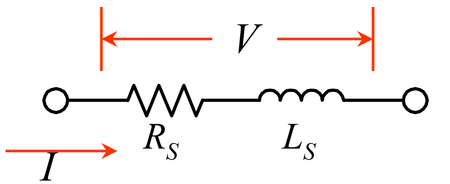
Typical $D \sim 10^{-4} – 0.1$

Capacitance series circuit: $$D = \frac{R_s}{X_s} = \omega C_s R_s$$

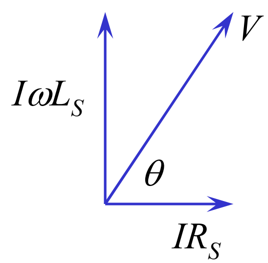
Inductor and Capacitor

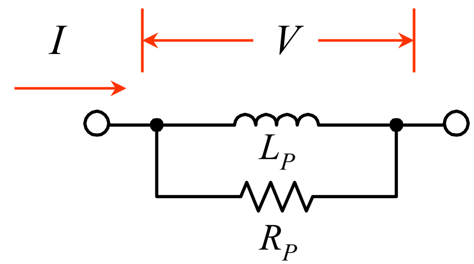
$$L_S = \frac{R_P^2}{R_P^2 + \omega^2 L_P^2} \cdot L_P$$

$$R_S = \frac{\omega^2 L_P^2}{R_P^2 + \omega^2 L_P^2} \cdot R_P$$



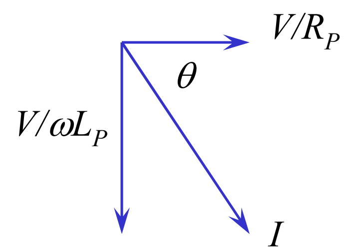
$$Q = \frac{\omega L_S}{R_S}$$





$$L_P = \frac{R_S^2 + \omega^2 L_S^2}{\omega^2 L_S^2} \cdot L_S$$

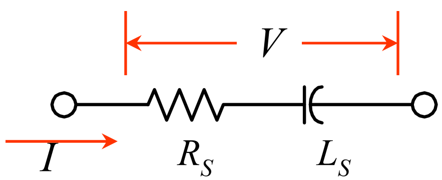
$$R_P = \frac{R_S^2 + \omega^2 L_S^2}{R_S^2} \cdot R_S$$



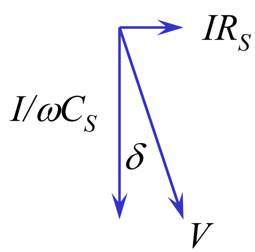
$$Q = \frac{R_P}{\omega L_P}$$

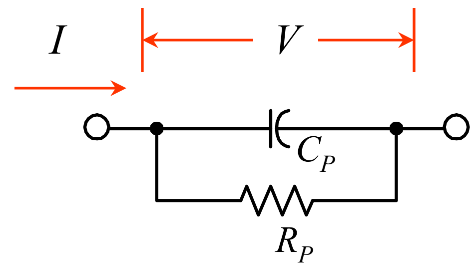
$$C_S = \frac{1 + \omega^2 C_P^2 R_P^2}{\omega^2 C_P^2 R_P^2} \cdot C_P$$

$$R_S = \frac{1}{1 + \omega^2 C_P^2 R_P^2} \cdot R_P$$



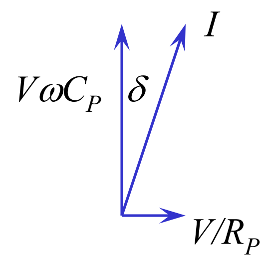
$$D = \omega C_S R_S$$





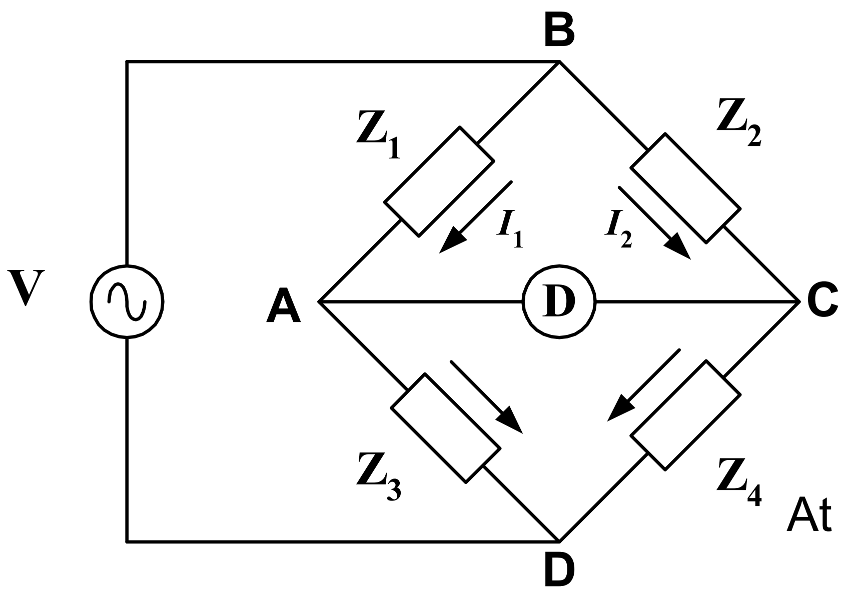
$$C_P = \frac{1}{1 + \omega^2 C_S^2 R_S^2} \cdot C_S$$

$$R_P = \frac{1 + \omega^2 C_S^2 R_S^2}{\omega^2 C_S^2 R_S^2} \cdot R_S$$



$$D = \frac{1}{\omega C_P R_P}$$

AC Bridge: Balance Condition



- all four arms are considered as impedance (frequency dependent components)
- The detector is an ac responding device: headphone, ac meter
- Source: an ac voltage at desired frequency

$\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$ and $\mathbf{Z}_4$ are the impedance of bridge arms

At balance point: $\mathbf{E}_{BA} = \mathbf{E}_{BC}$ or $\mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2$

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_3} \text{ and } \mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z}_2 + \mathbf{Z}_4}$$

General Form of the ac Bridge

Complex Form: $\boxed{\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3}$

Polar Form:

$$\boxed{Z_1Z_4\left(\angle\theta_1 + \angle\theta_4\right) = Z_2Z_3\left(\angle\theta_2 + \angle\theta_3\right)}$$

Magnitude balance: $\boxed{Z_1Z_4 = Z_2Z_3}$

Phase balance: $\boxed{\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3}$

Example The impedance of the basic ac bridge are given as follows:

$\mathbf{Z}_1 = 100\ \Omega\ \angle 80^{\circ}$ (inductive impedance)

$\mathbf{Z}_3 = 400\ \angle 30^{\circ}\Omega$ (inductive impedance)

$\mathbf{Z}_2 = 250\ \Omega$ (pure resistance)

$\mathbf{Z}_4 =$ unknown

Determine the constants of the unknown arm.

SOLUTION The first condition for bridge balance requires that

$$Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{250 \times 400}{100} = 1,000\ \Omega$$

The second condition for bridge balance requires that the sum of the phase angles of opposite arms be equal, therefore

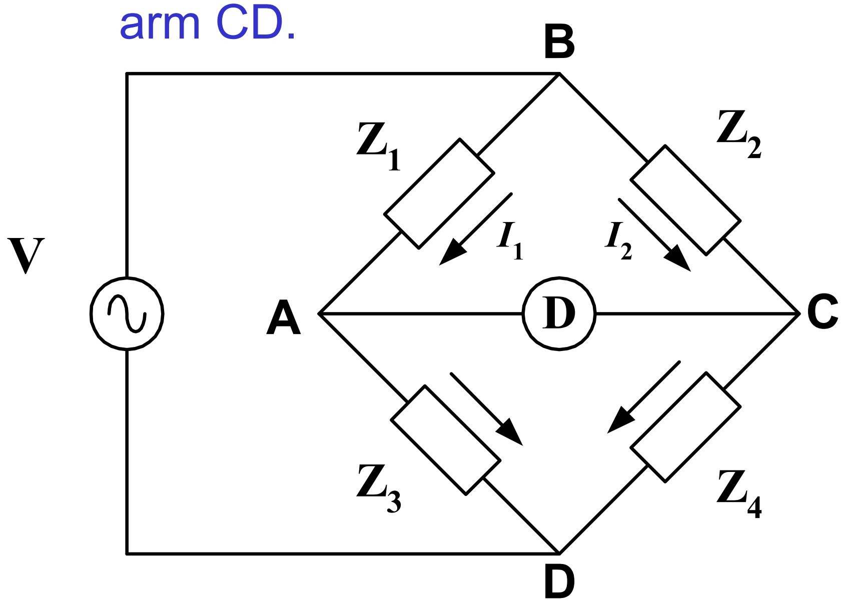
$$\angle\theta_4 = \angle\theta_2 + \angle\theta_3 - \angle\theta_1 = 0 + 30 - 80 = -50^{\circ}$$

Hence the unknown impedance $\mathbf{Z}_4$ can be written in polar form as

$$\mathbf{Z}_4 = 1,000\ \Omega\ \angle -50^{\circ}$$

Indicating that we are dealing with a capacitive element, possibly consisting of a series combination of at resistor and a capacitor.

Example an ac bridge is in balance with the following constants: arm AB, R = 200 Ω in series with L = 15.9 mH R; arm BC, R = 300 Ω in series with C = 0.265 μF; arm CD, unknown; arm DA, = 450 Ω. The oscillator frequency is 1 kHz. Find the constants of arm CD.



SOLUTION

$$\mathbf{Z}_1 = R + j\omega L = 200 + j100\ \Omega$$
$$\mathbf{Z}_2 = R + 1/j\omega C = 300 - j600\ \Omega$$
$$\mathbf{Z}_3 = R = 450\ \Omega$$
$$\mathbf{Z}_4 = \text{unknown}$$

The general equation for bridge balance states that $\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$

$$\mathbf{Z}_4 = \frac{\mathbf{Z}_2\mathbf{Z}_3}{\mathbf{Z}_1} = \frac{450 \times (200 + j100)}{(300 - j600)} = j150\ \Omega$$

This result indicates that $\mathbf{Z}_4$ is a pure inductance with an inductive reactance of 150 Ω at at frequency of 1kHz. Since the inductive reactance X_L = $2\pi fL$, we solve for L and obtain L = 23.9 mH

Comparison Bridge: Capacitance

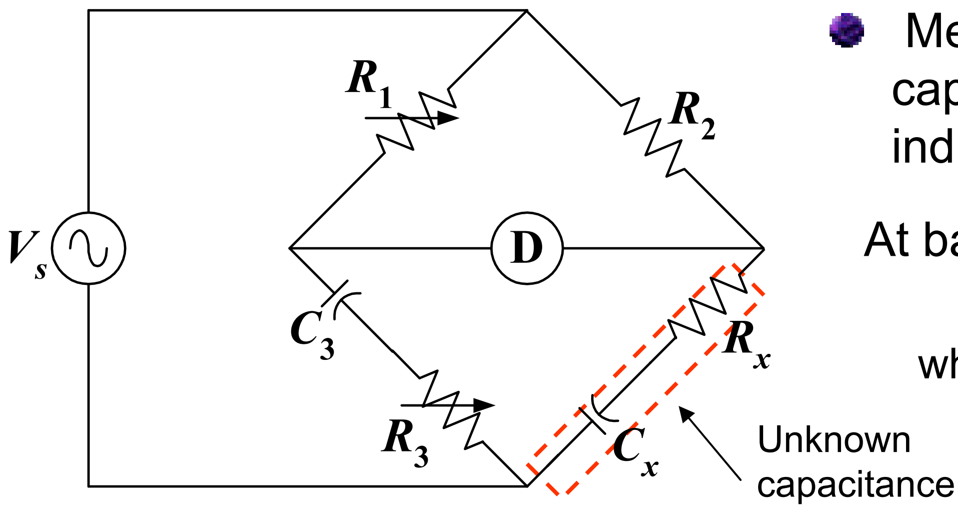


Diagram of Capacitance Comparison Bridge

- Measure an unknown inductance or capacitance by comparing with it with a known inductance or capacitance.

At balance point: $\mathbf{Z}_1\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3$

where $\mathbf{Z}_1 = R_1; \mathbf{Z}_2 = R_2;$ and $\mathbf{Z}_3 = R_3 + \dfrac{1}{j\omega C_3}$

$$R_1\left(R_x + \frac{1}{j\omega C_x}\right) = R_2\left(R_3 + \frac{1}{j\omega C_3}\right)$$

Separation of the real and imaginary terms yields: $\boxed{R_x = \dfrac{R_2 R_3}{R_1}}$ and $\boxed{C_x = C_3\dfrac{R_1}{R_2}}$

- Frequency independent
- To satisfy both balance conditions, the bridge must contain two variable elements in its configuration.

Comparison Bridge: Inductance

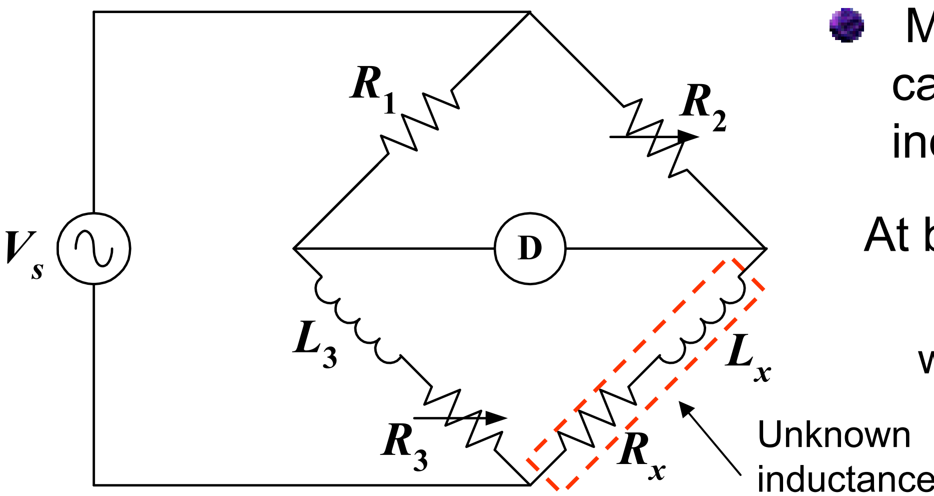


Diagram of Inductance Comparison Bridge

- Measure an unknown inductance or capacitance by comparing with it with a known inductance or capacitance.

At balance point: $\mathbf{Z}_1\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3$

where $\mathbf{Z}_1 = R_1; \mathbf{Z}_2 = R_2;$ and $\mathbf{Z}_3 = R_3 + j\omega L_3$

$$R_1\left(R_x + j\omega L_x\right) = R_2\left(R_S + j\omega L_S\right)$$

Separation of the real and imaginary terms yields: $\boxed{R_x = \dfrac{R_2 R_3}{R_1}}$ and $\boxed{L_x = L_3 \dfrac{R_2}{R_1}}$

- Frequency independent
- To satisfy both balance conditions, the bridge must contain two variable elements in its configuration.

Maxwell Bridge

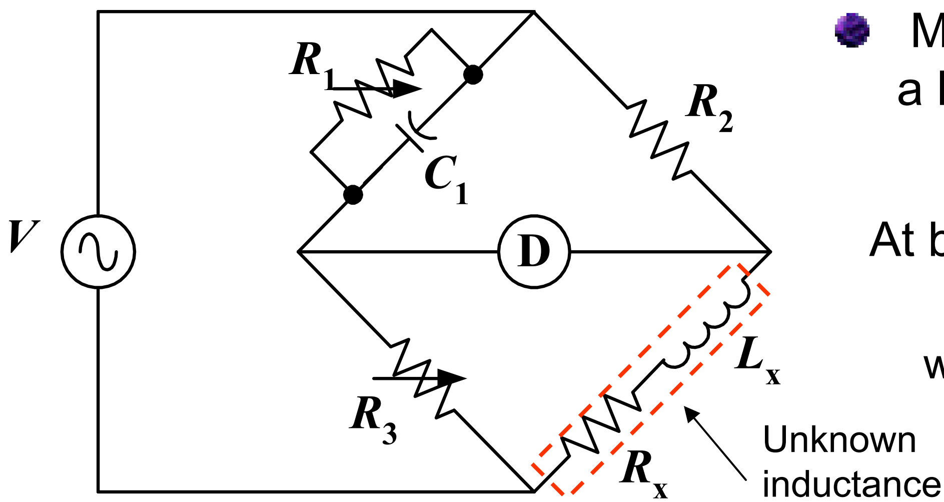


Diagram of Maxwell Bridge

- Measure an unknown inductance in terms of a known capacitance

At balance point: $\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3\mathbf{Y}_1$

where $\mathbf{Z}_2 = R_2;\ \mathbf{Z}_3 = R_3;\ \text{and}\ \mathbf{Y}_1 = \dfrac{1}{R_1} + j\omega C_1$

$$\mathbf{Z}_x = R_x + j\omega L_x = R_2R_3\left(\frac{1}{R_1} + j\omega C_1\right)$$

Separation of the real and imaginary terms yields: $\boxed{R_x = \dfrac{R_2R_3}{R_1}}$ and $\boxed{L_x = R_2R_3C_1}$

- Frequency independent
- Suitable for Medium Q coil (1-10), impractical for high Q coil: since R_1 will be very large.

Hay Bridge

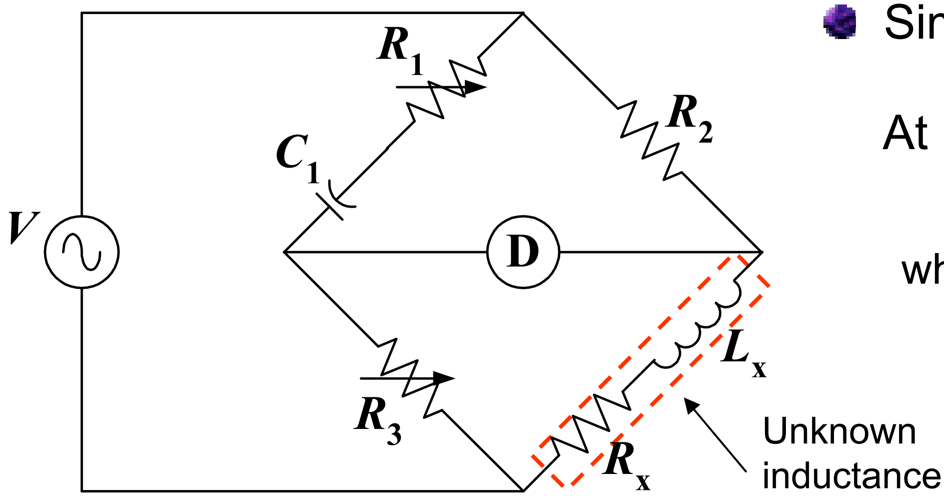


Diagram of Hay Bridge

- Similar to Maxwell bridge: but R_1 series with C_1

At balance point: $$\mathbf{Z}_1\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3$$

where $\mathbf{Z}_1 = R_1 - \dfrac{j}{\omega C_1}$; $\mathbf{Z}_2 = R_2$; and $\mathbf{Z}_3 = R_3$

$$\left(R_1 + \frac{1}{j\omega C_1}\right)\left(R_x + j\omega L_x\right) = R_2 R_3$$

which expands to $$R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad \text{..........(1)}$$

$$\frac{R_x}{\omega C_1} = \omega L_x R_1 \quad \text{..........(2)}$$

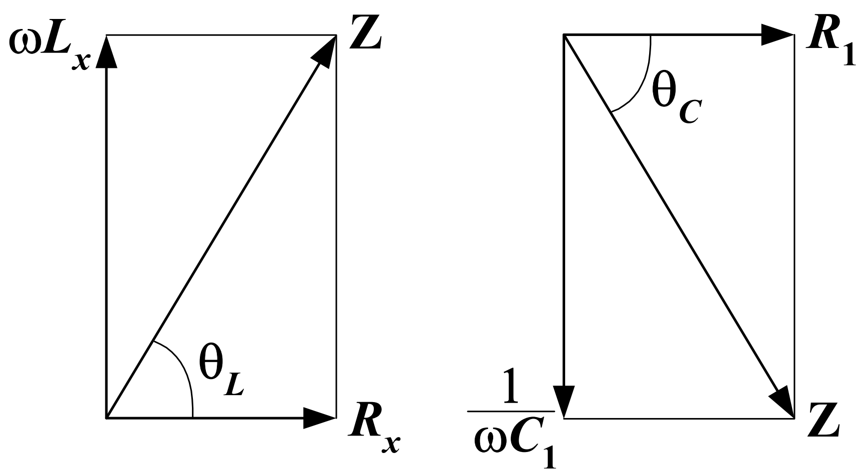
Solve the above equations simultaneously

Hay Bridge: continues

$$\boxed{R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 C_1^2 R_1^2}}$$

and

$$\boxed{L_x = \frac{R_2 R_3 C_1}{1+\omega^2 C_1^2 R_1^2}}$$



Phasor diagram of arm 4 and 1

$$\tan\theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q$$

$$\tan\theta_C = \frac{X_C}{R} = \frac{1}{\omega C_1 R_1}$$

$$\boxed{\tan\theta_L = \tan\theta_C \text{ or } Q = \frac{1}{\omega C_1 R_1}}$$

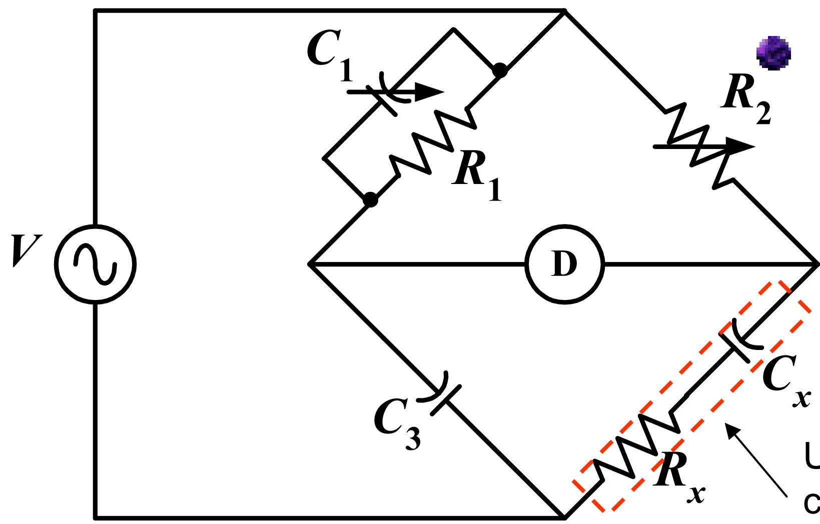
Thus, L_x can be rewritten as

$$L_x = \frac{R_2 R_3 C_1}{1+(1/Q^2)}$$

For high Q coil (> 10), the term $(1/Q)^2$ can be neglected

$$\boxed{L_x \approx R_2 R_3 C_1}$$

Schering Bridge



- Used extensively for the measurement of capacitance and the quality of capacitor in term of D

At balance point: $\mathbf{Z}_x = \mathbf{Z}_2\mathbf{Z}_3\mathbf{Y}_1$

where $\mathbf{Z}_2 = R_2;\ \mathbf{Z}_3 = \dfrac{1}{j\omega C_3};$ and $\mathbf{Y}_1 = \dfrac{1}{R_1} + j\omega C$

$$R_x - \frac{j}{\omega C_x} = R_2\left(\frac{-j}{\omega C_x}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

Diagram of Schering Bridge

which expands to $$R_x - \frac{j}{\omega C_x} = \frac{R_2 C_1}{C_3} - \frac{jR_2}{\omega C_3 R_1}$$

Separation of the real and imaginary terms yields: $\boxed{R_x = R_2\dfrac{C_1}{C_3}}$ and $\boxed{C_x = C_3\dfrac{R_1}{R_2}}$

Schering Bridge: continues

Dissipation factor of a series RC circuit: $$D = \frac{R_x}{X_x} = \omega R_x C_x$$

Dissipation factor tells us about the quality of a capacitor, how close the phase angle of the capacitor is to the ideal value of 90°

For Schering Bridge: $$\boxed{D = \omega R_x C_x = \omega R_1 C_1}$$

For Schering Bridge, R_1 is a fixed value, the dial of C_1 can be calibrated directly in D at one particular frequency

Wien Bridge

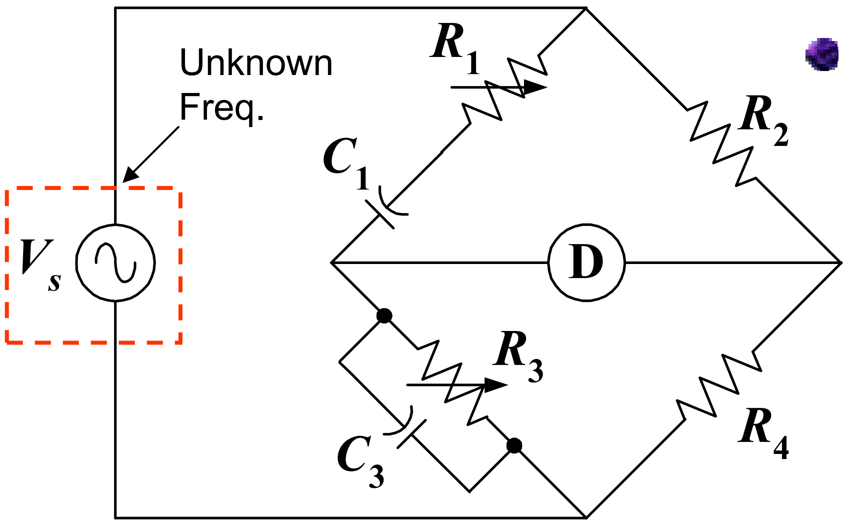


Diagram of Wien Bridge

- Measure frequency of the voltage source using series RC in one arm and parallel RC in the adjoining arm

At balance point: $\mathbf{Z}_2 = \mathbf{Z}_1\mathbf{Z}_4\mathbf{Y}_3$

$$\mathbf{Z}_1 = R_1 + \frac{1}{j\omega C_1}; \mathbf{Z}_2 = R_2; \mathbf{Y}_3 = \frac{1}{R_3} + j\omega C_3; \text{and } \mathbf{Z}_4 = R_4$$

$$R_2 = \left(R_1 - \frac{j}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + j\omega C_3\right)$$

which expands to $R_2 = \dfrac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \dfrac{jR_4}{\omega C_1 R_3} + \dfrac{R_4 C_3}{C_1}$

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad \text{----------- (1)}$$

$$\omega C_3 R_1 = \frac{1}{\omega C_1 R_3} \quad \text{----------- (2)}$$

Rearrange Eq. (2) gives $\boxed{f = \dfrac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}}}$

In most, Wien Bridge, $R_1 = R_3$ and $C_1 = C_3$

(1) $\rightarrow$ $\boxed{R_2 = 2R_4}$ (2) $\rightarrow$ $\boxed{f = \dfrac{1}{2\pi RC}}$

Wagner Ground Connection

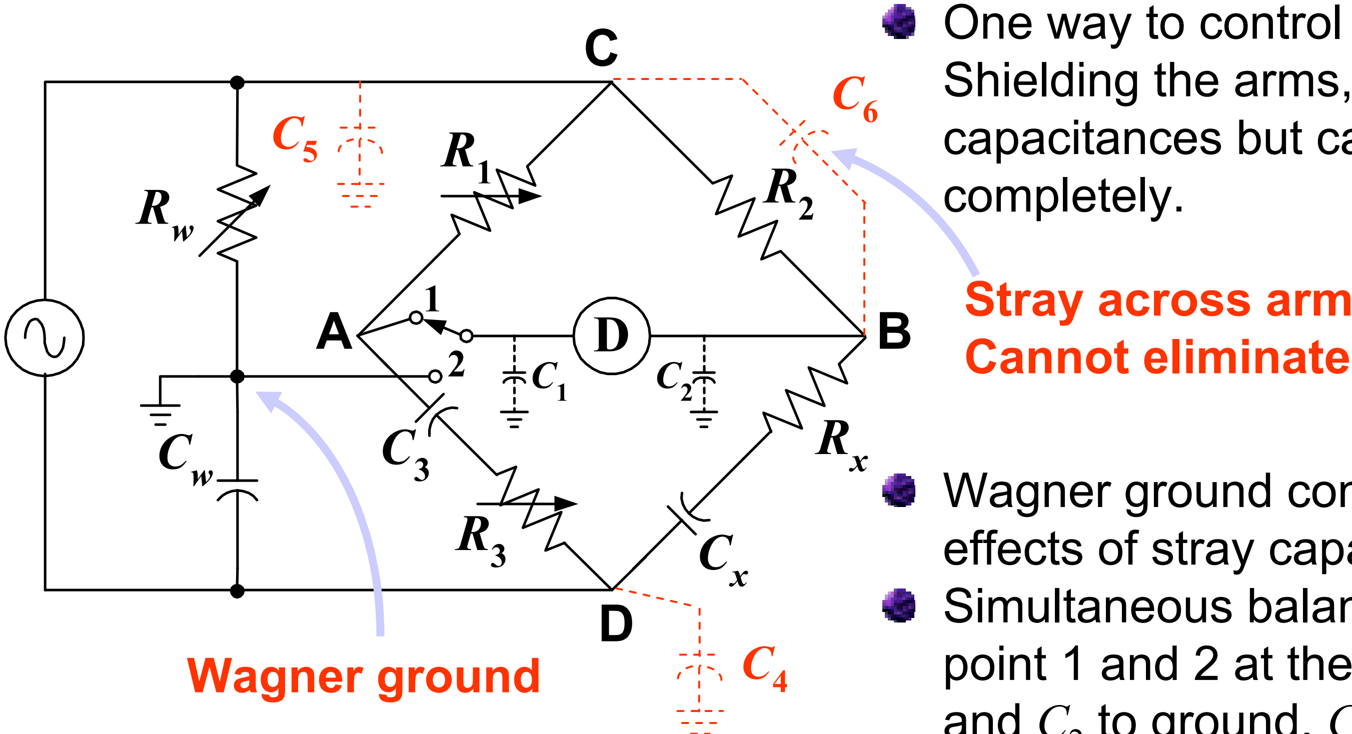


Diagram of Wagner ground

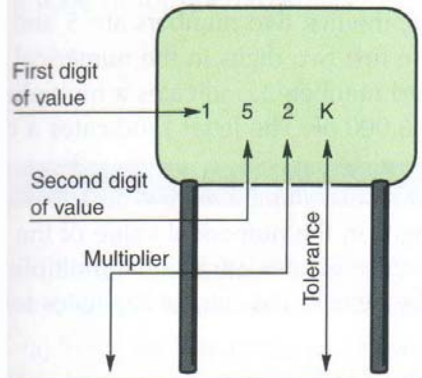
- One way to control stray capacitances is by Shielding the arms, reduce the effect of stray capacitances but cannot eliminate them completely.

- Wagner ground connection eliminates some effects of stray capacitances in a bridge circuit
- Simultaneous balance of both bridge makes the point 1 and 2 at the ground potential. (short C_1 and C_2 to ground, C_4 and C_5 are eliminated from detector circuit)
- The capacitance across the bridge arms e.g. C_6 cannot be eliminated by Wagner ground.

Capacitor Values

Ceramic Capacitor

Film-Type Capacitors



Multiplier		Tolerance of Capacitor		
For the Number	Multiplier	Letter	10 pF or Less	Over 10 pF
0 1	1 10	B C	±0.1 pF ±0.25 pF	
2 3	100 1,000	D F	±0.5 pF ±1.0 pF	±1%
4 5	10,000 100,000	G H	±2.0 pF	±2% ±3%
8	0.01	J K		±5% ±10%
9	0.1	M		±20%

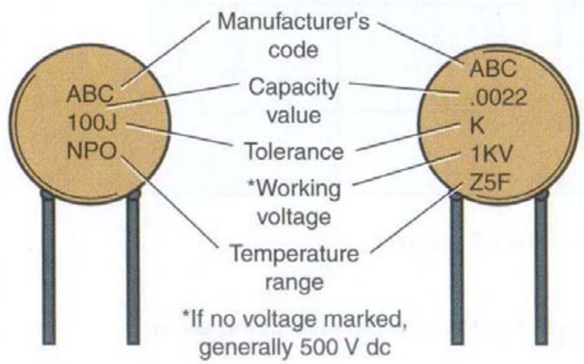
Examples:
152K = 15 × 100 = 1500 pF or 0.0015 μF, ±10%
759J = 75 × 0.1 = 7.5 pF, ±5%

Note: The letter R may be used at times to signify a decimal point, as in 2R2 = 2.2 (pF or μF).

Capacitor Values

Film Capacitor

Ceramic Disk Capacitors



Typical Ceramic Disk Capacitor Markings

Z 5 F 1 0 0 J

Low Temp.	Letter Symbol	High Temp.	Numerical Symbol	Max. Capacitance Change over Temp. Range	Letter Symbol
+10°C	Z	+45°C	2	+1.0%	A
−30°C	Y	+65°C	4	±1.5%	B
−55°C	X	+85°C	5	±1.1%	C
		+105°C	6	±3.3%	D
		+125°C	7	±4.7%	E
				±7.5%	F
				±10.0%	P
				±15.0%	R
				±22.0%	S
				+22%, −33%	T
				+22%, −56%	U
				+22%, −82%	V

Temperature Range Identification of Ceramic Disk Capacitors

1st & 2nd Fig. of Capacitance	Multiplier	Numerical Symbol	Tolerance on Capacitance	Letter Symbol
	1	0		
	10	1		
	100	2	±5%	J
	1,000	3	±10%	K
	10,000	4	±20%	M
	100,000	5	+100%, −0%	P
		—	+80%, −20%	Z
		—		
	0.01	8		
	0.1	9		

Capacity Value and Tolerance of Ceramic Disk Capacitors

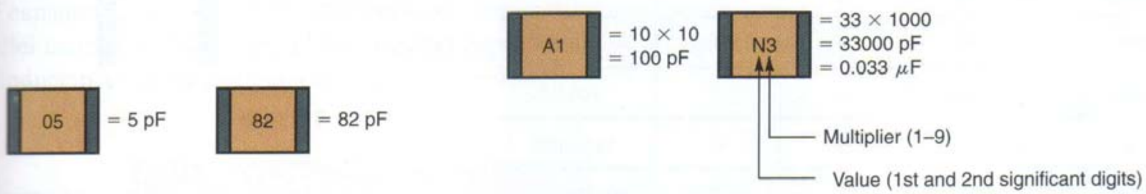
Capacitor Values

Chip Capacitor

Alternate Two-Place Code

• Values below 100 pF—Value read directly

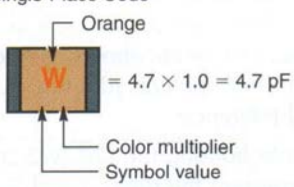
• Values 100 pF and above—Letter/number code



Value (24 Value Symbols)—Uppercase Letters Only					Multiplier
A-10 B-11 C-12 D-13 E-15	F-16 G-18 H-20 J-22 K-24	L-27 M-30 N-33 P-36 Q-39	R-43 S-47 T-51 U-56 V-62	W-68 X-75 Y-82 Z-91	1 = × 10 2 = × 100 3 = × 1,000 4 = × 10,000 5 = × 100,000 etc.

18 Chip capacitor coding system.

Standard Single-Place Code



Examples: R (Green) = 3.3 × 100 = 330 pF
7 (Blue) = 8.2 × 1000 = 8200 pF

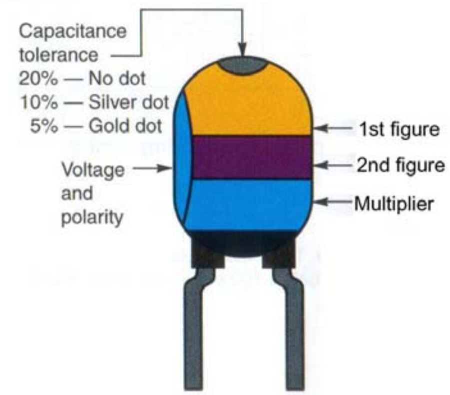
Value (24 Value Symbols)—Uppercase Letters and Numerals					Multiplier (Color)
A-1.0 B-1.1 C-1.2 D-1.3 E-1.5	H-1.6 I-1.8 J-2.0 K-2.2 L-2.4	N-2.7 O-3.0 R-3.3 S-3.6 T-3.9	V-4.3 W-4.7 X-5.1 Y-5.6 Z-6.2	3-6.8 4-7.5 7-8.2 9-9.1	Orange = × 1.0 Black = × 10 Green = × 100 Blue = × 1,000 Violet = × 10,000 Red = × 100,000

Capacitor Values

Tantalum Capacitor

Color	Rated Voltage	Capacitance in Picofarads 1st Figure	Capacitance in Picofarads 2nd Figure	Multiplier
Black	4	0	0	—
Brown	6	1	1	—
Red	10	2	2	—
Orange	15	3	3	—
Yellow	20	4	4	10,000
Green	25	5	5	100,000
Blue	35	6	6	1,000,000
Violet	50	7	7	10,000,000
Gray	—	8	8	—
White	3	9	9	—

Dipped Tantalum Capacitors



Capacitor Values

Chip Capacitor

Value (33 Value Symbols)—Upper and Lowercase Letters					Multiplier
A-1.0	H-2.0	b-3.5	f-5.0	X-7.5	0 = × 1.0
B-1.1	J-2.2	P-3.6	T-5.1	t-8.0	1 = × 10
C-1.2	K-2.4	Q-3.9	U-5.6	Y-8.2	2 = × 100
D-1.3	a-2.5	d-4.0	m-6.0	y-9.0	3 = × 1,000
E-1.5	L-2.7	R-4.3	V-6.2	Z-9.1	4 = × 10,000
F-1.6	M-3.0	e-4.5	W-6.8		5 = × 100,000
G-1.8	N-3.3	S-4.7	n-7.0		etc.